

Annual Review:
Whispering-gallery mode resonators,
second harmonic generation,
and ultrafast lasers

Matt T. Simons

Department of Physics
College of William & Mary

Outline

- 1 Recap
- 2 Whispering-gallery progress
 - Experiment
 - Theory
- 3 Second harmonic generation at Rb λ
- 4 Ultrafast center
- 5 Future plans

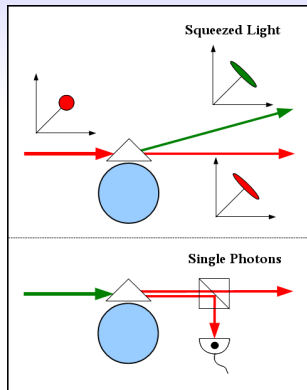
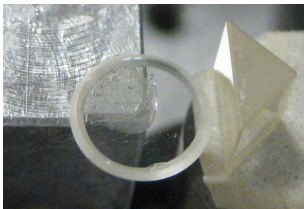
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Motivation

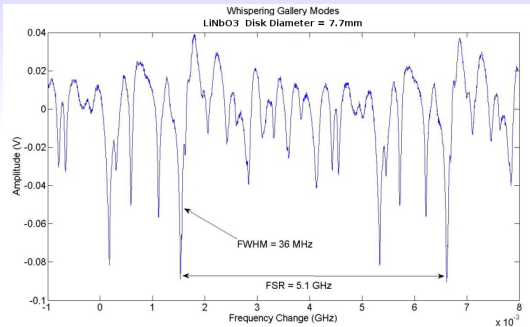
Develop a source of nonclassical light based on nonlinear processes using crystalline whispering-gallery mode resonators.

- Source of bright squeezed light
- Heralded single photon source



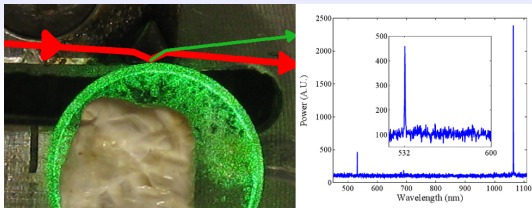
Previously . . .

- LiNbO₃ WGMRs
- Q-factor > 10⁷
- 1064nm → 532nm
- Hyper-Raman scattering



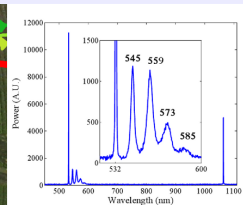
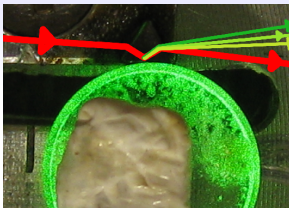
Previously . . .

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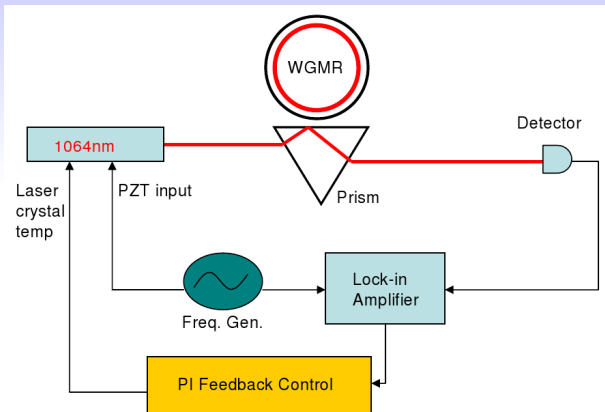
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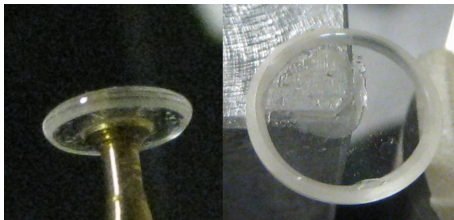
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1064nm Tunable laser



Annealing WGMR disk

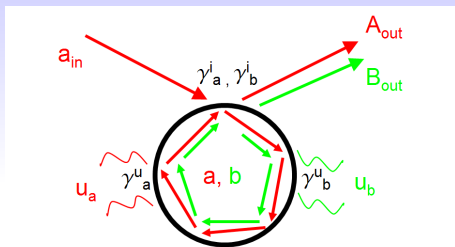
- 1 Polish disk with $0.1 \mu\text{m}$ diamond paper
- 2 Anneal 600° for 24 hrs.
- 3 Polish disk with $0.1 \mu\text{m}$ diamond paper
- 4 $Q = 3 \times 10^6 \rightarrow 8 \times 10^6$



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Theoretical model



The Hamiltonian inside the cavity is

$$H_{sys} = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \frac{\iota}{2} \hbar\epsilon (a^\dagger a^\dagger b - aab^\dagger) \quad (1)$$

Variance

Noise in the amplitude quadrature of the output field ($A_1^{out} = A_{out} + A_{out}^\dagger$) is calculated by the variance:

$$\text{Var}(A_1^{out}) = \langle |A_1^{out} - \langle A_1^{out} \rangle|^2 \rangle \quad (2)$$

$$A_1^{out} = \langle A_1^{out} \rangle + \delta A_1^{out} \quad (3)$$

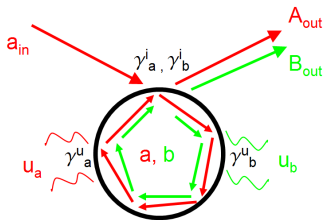
$$\text{Var}(A_1^{out}) = \langle |\delta A_1^{out}|^2 \rangle \quad (4)$$

Theory

With $\dot{x} = -\frac{i}{\hbar}[x, H]$ the intracavity fields change in time as:

$$\dot{a} = -i\omega_a a - \frac{1}{2}\gamma_a^{tot} a + \epsilon a^\dagger b + \sqrt{\gamma_a^i} a_{in} + \sqrt{\gamma_a^u} u_a \quad (5)$$

$$\dot{b} = -i\omega_b b - \frac{1}{2}\gamma_b^{tot} b - \frac{1}{2}\epsilon a a^\dagger + \sqrt{\gamma_b^i} b_{in} + \sqrt{\gamma_b^u} u_b \quad (6)$$



Theory

Assuming unseeded SHG ($\bar{b}_{in} = 0$), and approximating each field as $x = \langle x \rangle + \delta x$, the equations describing the fluctuation of the fields are

$$\dot{\delta a} = -\frac{1}{2}\gamma_a^{tot}\delta a + \epsilon\bar{a}^*\delta b + \epsilon\bar{b}\delta a^\dagger + \sqrt{\gamma_a^i}\delta a_{in} + \sqrt{\gamma_a^u}\delta u_a \quad (7)$$

$$\dot{\delta b} = -\frac{1}{2}\gamma_b^{tot}\delta b - \epsilon\bar{a}\delta a + \sqrt{\gamma_b^i}\delta b_{in} + \sqrt{\gamma_b^u}\delta u_b \quad (8)$$

$$\tilde{x}_c \equiv \begin{pmatrix} \tilde{\delta a} \\ \tilde{\delta a}^\dagger \\ \tilde{\delta b} \\ \tilde{\delta b}^\dagger \end{pmatrix}, \quad \tilde{x}_{in} \equiv \begin{pmatrix} \tilde{\delta a}_{in} \\ \tilde{\delta a}_{in}^\dagger \\ \tilde{\delta b}_{in} \\ \tilde{\delta b}_{in}^\dagger \end{pmatrix}, \quad \tilde{x}_u \equiv \begin{pmatrix} \tilde{\delta u}_a \\ \tilde{\delta u}_a^\dagger \\ \tilde{\delta u}_b \\ \tilde{\delta u}_b^\dagger \end{pmatrix} \quad (9)$$

such that the fluctuation equations can be expressed in matrix form:

$$i\Omega\tilde{x}_c = M_c\tilde{x}_c + M_{in}\tilde{x}_{in} + M_u\tilde{x}_u \quad (10)$$

Theory

$$\tilde{x}_c = (i\Omega I - M_c)^{-1} (M_{in}\tilde{x}_{in} + M_u\tilde{x}_u) \quad (11)$$

$$\tilde{x}_o \equiv \begin{pmatrix} \delta A_{out} \\ \delta A_{out}^\dagger \\ \delta B_{out} \\ \delta B_{out}^\dagger \end{pmatrix} = M_{in}\tilde{x}_c - \tilde{x}_{in} \quad (12)$$

$$\begin{aligned} \tilde{x}_o &= [M_{in} (i\Omega I - M_c)^{-1} M_{in} - I]\tilde{x}_{in} \\ &\quad + M_{in} (i\Omega I - M_c)^{-1} M_u\tilde{x}_u \end{aligned} \quad (13)$$

$$\delta A_1^{out} = \delta A_{out} + \delta A_{out}^\dagger \quad (14)$$

Theory

Solved for $\text{Var}(A_1^{\text{out}}) = \langle |\delta A_1^{\text{out}}|^2 \rangle$ as a function of

- WGMR quality factor Q

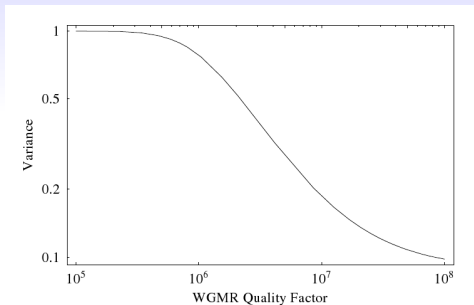


Figure: $f = 10$ MHz, $P_{in} = 500 \mu\text{W}$

Theory

Solved for $\text{Var}(A_1^{out}) = \langle |\delta A_1^{out}|^2 \rangle$ as a function of

- WGMR quality factor Q
- Input power P_{in}

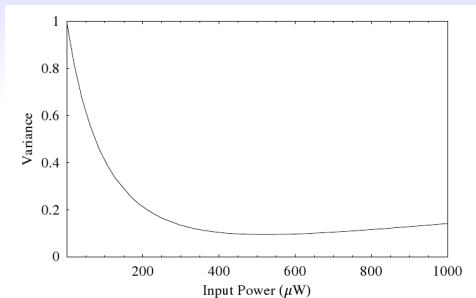


Figure: $Q = 10^8, f = 10 \text{ MHz}$

Theory

Solved for $\text{Var}(A_1^{out}) = \langle |\delta A_1^{out}|^2 \rangle$ as a function of

- WGMR quality factor Q
- Input power P_{in}
- Detection frequency f

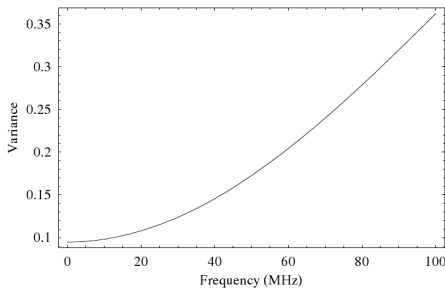


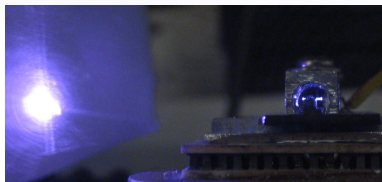
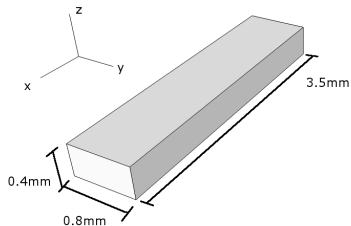
Figure: $Q = 10^8$, $P_{in} = 500 \mu\text{W}$

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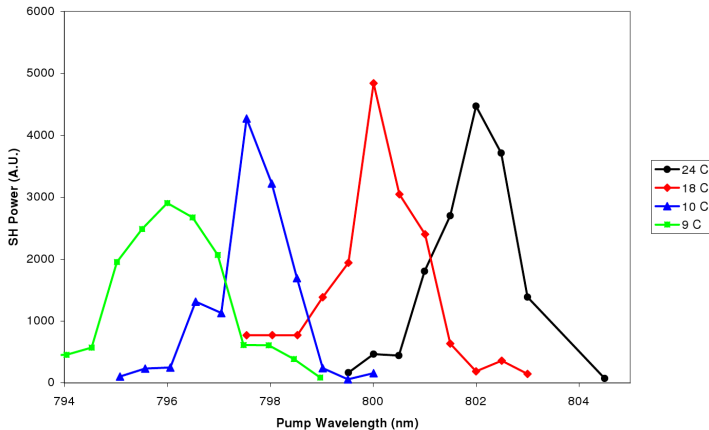
Potassium lithium niobate (KLN)

Second harmonic generation $800\text{ nm} \rightarrow 400\text{ nm}$
using natural phase matching



Potassium lithium niobate (KLN)

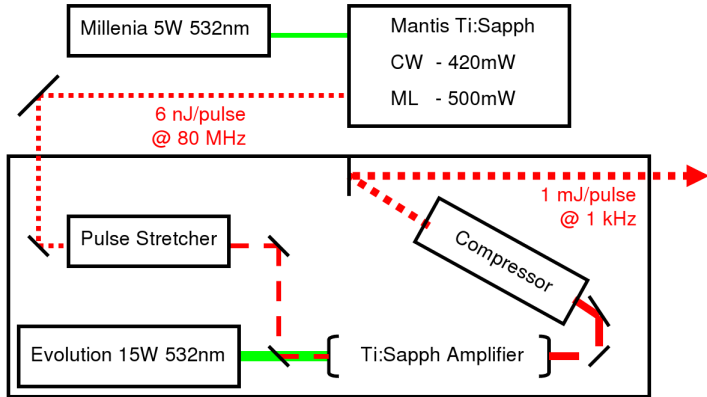
Second harmonic power vs. pump wavelength
for different KLN crystal temperatures



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Legend femtosecond laser system

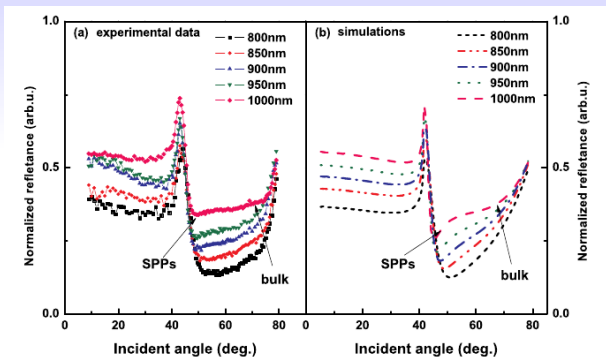


TOPAS

Traveling-wave optical parametric amplifier of super-fluorescence
Tunable from 470 nm to 1600 nm
INSERT GRAPH of WAVELENGTHS

Surface plasmon resonances

- Measured reflection from RuO₂ thin film
- Using TOPAS $\lambda = 800 \rightarrow 1000$ nm

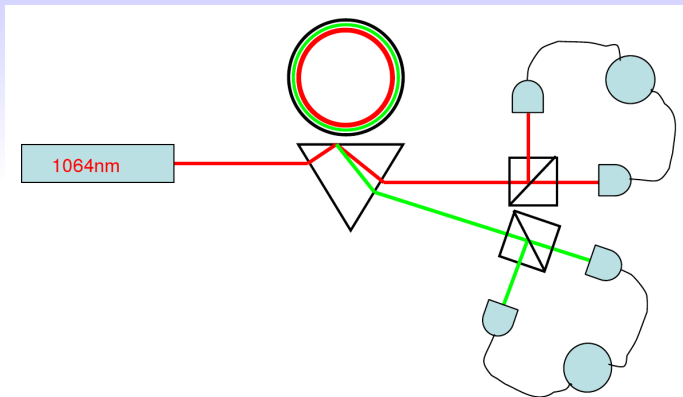


L. Wang, C. Clavero, K. Yang, E. Radue, M. T. Simons, I. Novikova, and R. A. Lukaszew. *Bulk and surface plasmon polariton excitation in RuO₂ for low-loss plasmonic applications in NIR.* Opt. Express 20, 8618-8628 (2012).

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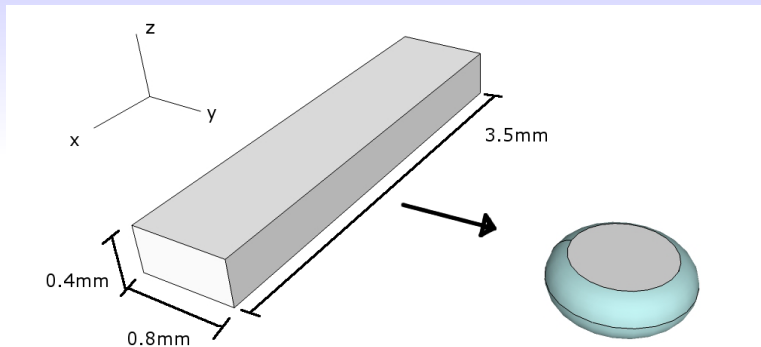
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Second harmonic squeezing



KLN Whispering-gallery disks

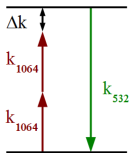
Demonstrate naturally-phase matched SHG from 795 \rightarrow 397 nm



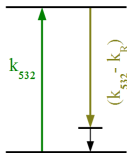
Demonstrate SHG squeezing at 795 nm

Hyper-Raman squeezing

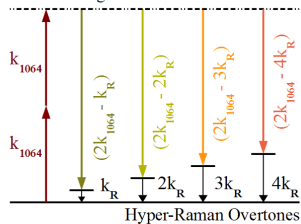
a. Second Harmonic Generation



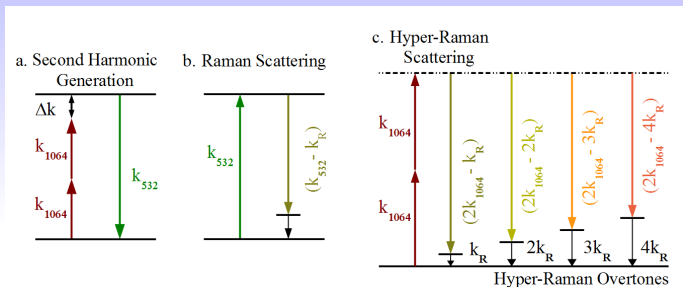
b. Raman Scattering



c. Hyper-Raman Scattering

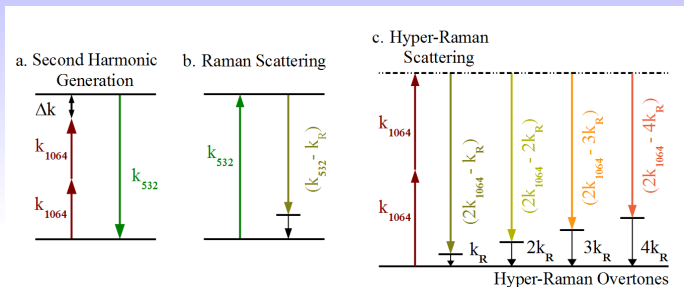


Hyper-Raman squeezing



- “In hyper-Raman scattering squeezing exists ... in the fundamental pump mode.”

Hyper-Raman squeezing



- “In hyper-Raman scattering squeezing exists ... in the fundamental pump mode.”
- “The Stokes mode in hyper-Raman scattering is squeezed when a fundamental mode with amplitude-squared squeezing propagates through a nonlinear medium.”

Thank you!

Questions?

Matrices

$$M_c \equiv \begin{pmatrix} -\frac{1}{2}\gamma_a^{tot} & \epsilon\bar{b} & \epsilon\bar{a}^* & 0 \\ \epsilon\bar{b}^* & -\frac{1}{2}\gamma_a^{tot} & 0 & \epsilon\bar{a} \\ -\epsilon\bar{a} & 0 & -\frac{1}{2}\gamma_b^{tot} & 0 \\ 0 & -\epsilon\bar{a}^* & 0 & -\frac{1}{2}\gamma_b^{tot} \end{pmatrix} \quad (15)$$

$$M_{in} \equiv \text{diag} \left(\sqrt{\gamma_a^i}, \sqrt{\gamma_a^i}, \sqrt{\gamma_b^i}, \sqrt{\gamma_b^i} \right) \quad (16)$$

$$M_u \equiv \text{diag} \left(\sqrt{\gamma_a^u}, \sqrt{\gamma_a^u}, \sqrt{\gamma_b^u}, \sqrt{\gamma_b^u} \right) \quad (17)$$

HRS Paper

add

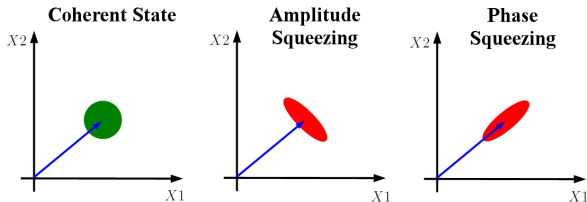
Squeezed light

Particle: position & momentum uncertainty relation:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (18)$$

Light: amplitude & phase uncertainty relation:

$$\Delta A \Delta \Phi \geq \frac{1}{2} \quad (19)$$

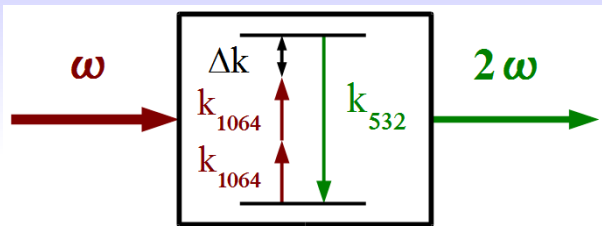


Motivation for squeezed light

- Sensitive measurements (LIGO, etc.)
- Quantum cryptography
- Quantum computing

Second harmonic generation

$$\vec{P} \sim \chi^{(1)} \vec{E} + \chi^{(2)} \vec{E}^2 + \dots \quad (20)$$



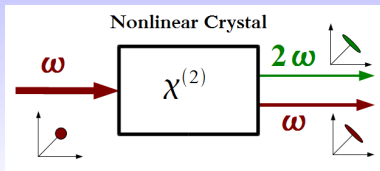
- Energy Conservation

$$\omega + \omega = 2\omega \quad (21)$$

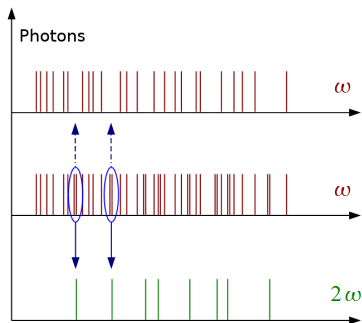
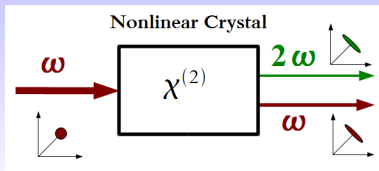
- Momentum conservation

$$\Delta k = k_{2\omega} - 2k_{\omega} = \frac{2\omega}{c} (n(2\omega) - n(\omega)) \quad (22)$$

Squeezed light



Squeezed light

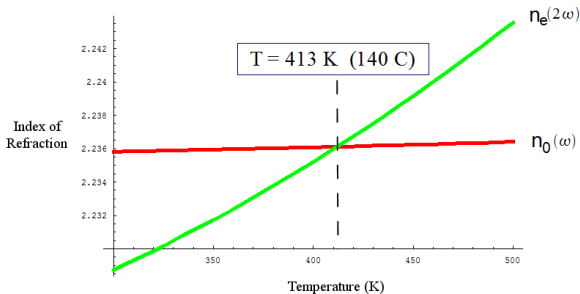


Second harmonic generation

- Momentum conservation

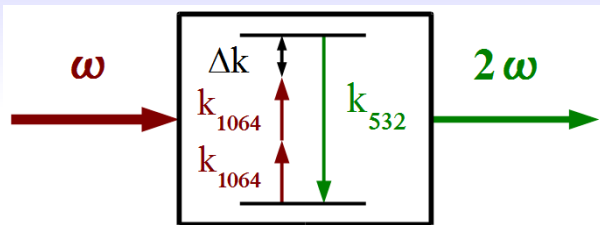
$$\Delta k = k_{2\omega} - 2k_{\omega} = \frac{2\omega}{c}(n(2\omega) - n(\omega))$$

- $\Delta k \rightarrow 0$ Phase-matching



Second harmonic generation

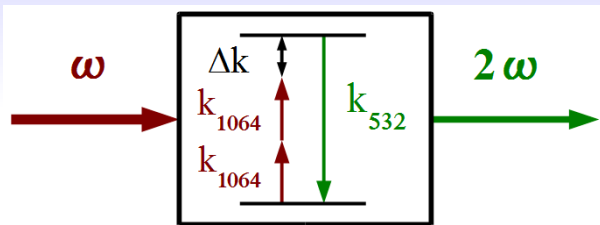
$$\vec{P} \sim \chi^{(1)}\vec{E} + \chi^{(2)}\vec{E}^2 + \dots$$



Intensity dependent process

Second harmonic generation

$$\vec{P} \sim \chi^{(1)}\vec{E} + \chi^{(2)}\vec{E}^2 + \dots$$

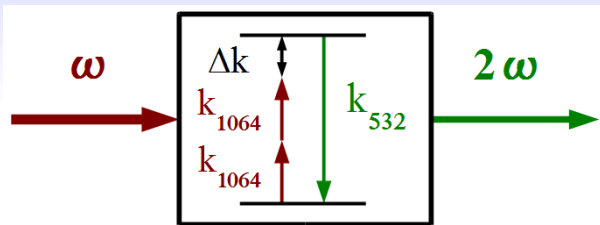


Intensity dependent process

- High-power pump laser

Second harmonic generation

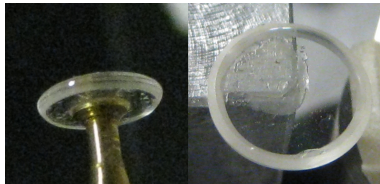
$$\vec{P} \sim \chi^{(1)}\vec{E} + \chi^{(2)}\vec{E}^2 + \dots$$



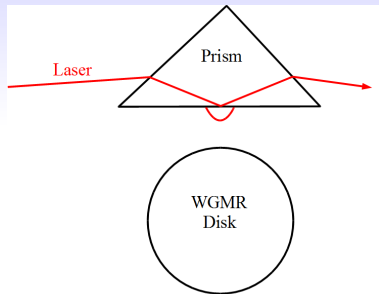
Intensity dependent process

- High-power pump laser
- High-quality cavity

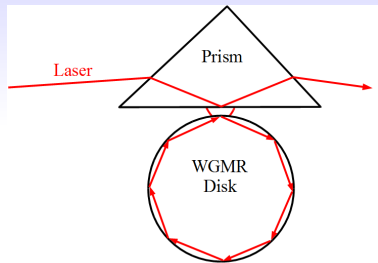
Whispering-gallery mode resonators (WGMRs)



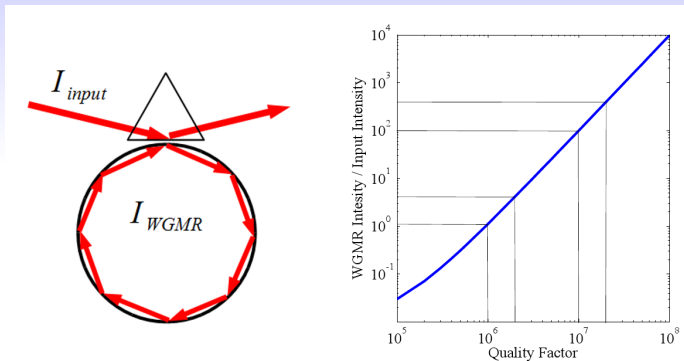
Coupling to whispering-gallery modes



Coupling to whispering-gallery modes



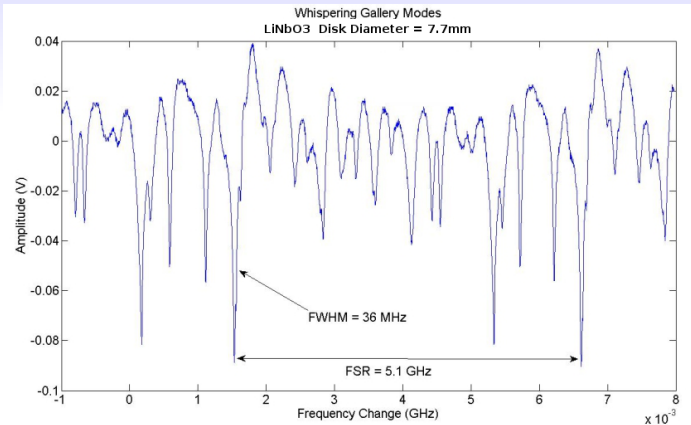
Whispering-gallery mode resonators



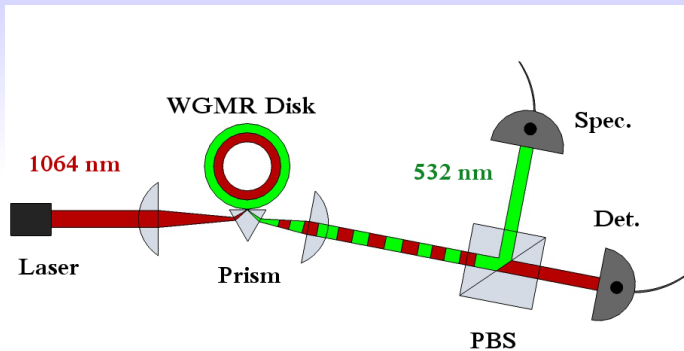
Coupling to whispering-gallery modes

Frequency scanned output from our LiNbO_3 WGMR disk near 795nm

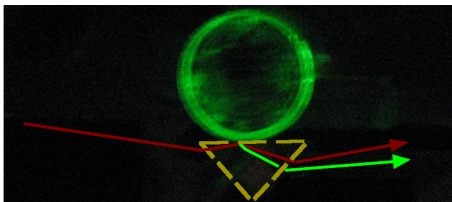
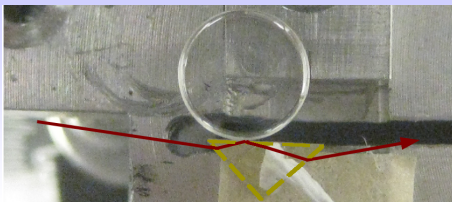
Q-factor of $Q = 10^7$



Second harmonic generation



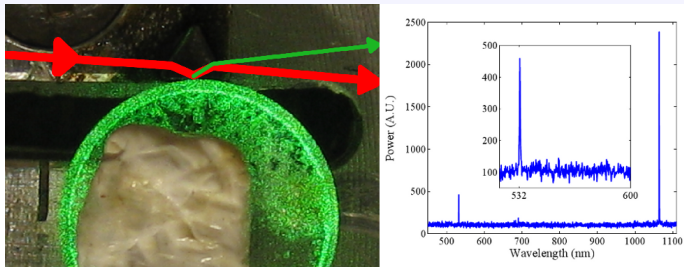
Second harmonic generation



Second harmonic generation

Input 1064 nm Power = 11 mW

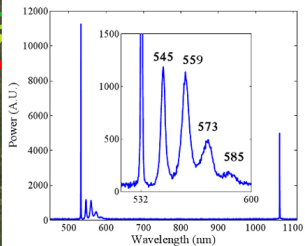
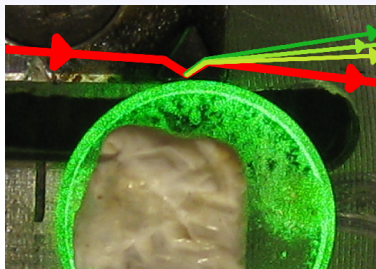
T = 26 °C



Raman scattering of the second harmonic

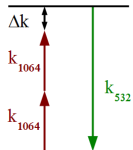
Input 1064 nm Power = 650 mW

T = 26 °C

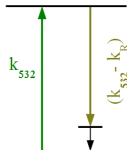


Hyper-Raman scattering

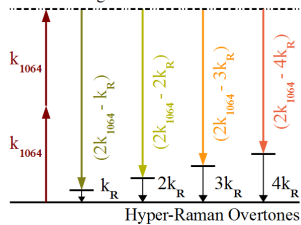
a. Second Harmonic Generation



b. Raman Scattering



c. Hyper-Raman Scattering



Summary

- Demonstrated whispering-gallery mode disks
- Observed SHG in WGMR disks
- Predict bright squeezed light
- Observed hyper-Raman scattering

Acknowledgements

