

Improving Optical Gyroscope Sensitivity Using a Fast Light Regime

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1 Abstract

Optical gyroscopes provide precision measurements of angular velocity for navigational systems and can be used to make precision measurements of the earth's rotation. Several-fold improvements to optical gyroscope sensitivity are predicted for fast light regimes ($n_g < 1$). The feasibility of these improvements will be evaluated using rubidium and with an engineered negative dispersion to achieve fast light, and a suitably high gain to allow for lasing within the cavity.

2 Introduction

2.1 Motivation

Historically gyroscopes have been large top like devices that rely on the conservation of Momentum to maintain there original spatial orientation. In the 1970s,optical gyroscopes became popular for certain navigational applications [2] . Optical gyroscopes rotate freely with the device and provide measurements of angular velocity rather than providing a fixed reference for spatial orientation.

In aviation systems Gyroscopes are used to stabilize aircraft trajectories. In environments where external position references such as GPS are unavailable, gyroscopes can be used in concert with other sensors such as accelerometers to employ a navigational method called inertial navigation [1]. Inertial navigation uses gyroscopes and accelerometers to calculate find the angular velocity, and linear acceleration of the navigating vehicle. A computer will then use integration from a known starting point to calculating position, heading and speed [1]. Optical gyroscopes, because they provide precise measurements of the angular velocity have been used to make very precise measurements of the rotation rate of the earth.

2.2 Background

2.2.1 The Sagnac effect

When light is propagate through a ring rotating at angular velocity Ω the light will need to either travel slightly further to catch up with its original starting position, or it will meet its starting position earlier than would be expected in a non-rotating cavity (depending on whether the direction of the light and the direction of the rotation are the same or opposite). In a ring cavity therefore the apparent cavity length will be altered in the different directions. according to the equation

$$\Delta L = \frac{2\pi R^2 \Omega}{c + R\Omega} \quad (1)$$

Where ΔL is the change in path length, R is the radius of the cavity, c is the speed of light, and Ω is the gyroscope angular velocity. The resonance condition for optical cavities ($n\lambda=L$) requires that the change in the optical path length will split the resonant frequencies based on the direction of propagation. The new split resonant frequencies can be calculated in the limit where $R\Omega \ll c$

$$f'_{\pm} = f_0 \pm \frac{A\Omega}{\lambda P} \quad (2)$$

Where $f'_{/pm}$ are the new resonant frequencies, A is the area of the gyroscope cavity Ω is the angular velocity of the gyroscope, P is the perimeter of the ring cavity, and λ is the wavelength of light in the cavity. Therefore the splitting between the two new resonances can be expressed by the formula

$$\Delta f_{empty} = \frac{2A\Omega}{\lambda P} \quad (3)$$

where Δf_{empty} is the frequency splitting between the resonances of counter propagating beams in a rotating cavity under the assumed condition "empty cavity conditions", that is to say deviations of the index of refraction from vacuum are negligible as is optical dispersion (assumptions that will be reconsidered in 2.2.3). Optical gyroscopes function by measuring the splitting between the resonant frequencies of counter propagating beams in a ring cavity to calculate angular velocity.

2.2.2 Laser Gyroscopes

This work considers a special class of optical gyroscopes: Laser Gyroscopes. The fundamental concept behind laser gyroscopes is fairly simple, A laser gyroscope is just an optical gyroscope that is also a laser. In its most simple form a laser is a gain medium in a cavity. The gain medium amplifies incoming light, the amplified light is then cycled back into the amplifier which causes a positive feedback loop which will cause the total power of the light in the cavity to saturate due to the limited power output of the gain medium, some of the power is allowed to leak out through a partially transmissive cavity boundary. The function of the laser cavity is not just to provide positive feedback, it also restricts the wavelength of the laser light to resonant frequency of the cavity. Laser light is therefore reasonably close to monochromatic and is at a frequency connected to the size of the cavity the gain medium is in.

A laser gyroscope consists of a gain medium inside of a ring cavity thus creating a laser will generate two counter propagating beams within the cavity which have frequencies locked to the resonances of the rotating cavity. In a laser gyroscope the frequencies of the fields inside of the cavity will be matched to the apparent path lengths of the cavity in counter propagating directions. Laser gyroscopes use these internally generated fields rather than some externally injected light source to find the resonant frequencies of the rotating cavity.

2.2.3 Fast Light

The theoretical consideration of the optical gyroscope presented earlier (2.2.1) was based on the assumption that the medium in the cavity was vacuum and was absent of any dispersions. The contributions of refractive index and dispersion are far from negligible. The resonance condition for optical cavities is

not dependent on the absolute length of the cavity but instead on the optical path length of the cavity. Optical path length is concerned with phase shifts within light waves therefore the speed of light in that specific medium has a significant effect on path length. Mathematically, the optical path length is the product of the index of refraction and the absolute path length. the resonance condition for cavities then becomes

$$m\lambda = nL \quad (4)$$

where m is an integer, λ is the wavelength, L is the round trip length of the cavity and n is the index of refraction. Dispersions further complicate the matter, because in a dispersive material the speed of light, and therefore the index of refraction, have a dependency on frequency. Dispersion terms represent the first derivative of this dependency. The effects of dispersion on refractive index are shown in this first order approximation of index of refraction in a dispersive medium.

$$n_g = n_0 + f \frac{\partial n}{\partial f} \quad (5)$$

where n_0 is the refractive index, f is the frequency of light, and $\frac{\partial n}{\partial f}$ is the dispersion. The group index is related to the group velocity

$$v_g = \frac{c}{n_g} \quad (6)$$

Which describes the velocity at which the envelopes of an optical signal propagates. In contrast to the phase velocity, the group velocity can be superluminal under certain conditions as it does not necessarily carry information. This can be achieved by having a sufficiently negative dispersion such that $n_g < 1$, light propagating under this condition is referred to as fast light.

Theory predicts that the changes in the resonant frequencies due to a change in length is dependent on group velocity. The frequency splitting in a dispersive cavity can be related to the frequency splitting in the empty cavity by

$$\Delta f_{dispersive} = \frac{\Delta f_{empty}}{n_g} \quad (7)$$

In order to achieve the negative dispersion required to support fast light regimes the medium will have to display certain optical properties. The Kramers-Kronig Relations describe the relationship between the index of refraction in a medium and the absorption of a medium. In order for strong negative dispersions to exist in a medium they must have an absorption like feature in the region of strong negative dispersion. In laser gyroscopes this tends to be problematic because laser gyroscopes

require strong gain (at least exceeding unity) and strong absorption in the fast light region would mean that lasing will likely not be supported by the system. The ideal atomic system for Generating fast light in medium will be strong amplification with a small dip that does not go below unity gain. Previous efforts have demonstrated that optical gain and steep dispersion can coexist in a particular N-bar four wave mixing scheme in the ^{87}Rb atomic structure Two pump fields tuned to 795nm (Ω_1) and 780nm (Ω_2) interact with the rubidium atom and generate two new fields, each detuned by 6.8GHz from each of the pump fields.(See Figure.1) ^{2,3} The strong gain and steep dispersion make this particular scheme

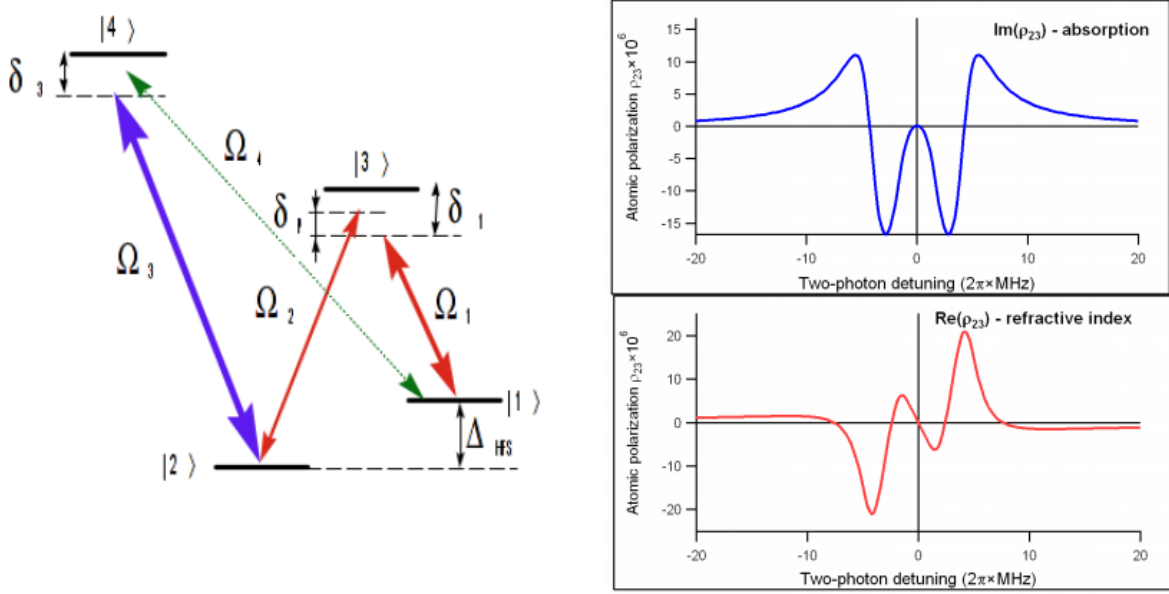


Figure 1: (Right) A simplified N-bar four wave mixing scheme to be tested as a medium provide gain for lasing as well as the conditions for fast light. (left) Predicted absorption and refractive index relations for this simplified four wave mixing scheme.

a good candidate for use in laser Gyroscope applications.

3 Experiment

3.1 Optical Apparatus

The gyroscope cavity used in this project consists of a square ring cavity containing a Rubidium cell used to achieve four wave mixing to provide gain and strong negative dispersion. Four-wave mixing is induced in the Rubidium cell by injecting two pump fields tuned to the D1 and D2 Resonances of Rubidium (794.7nm and 780nm respectively). The pumps are injected into the rubidium cell. two of the sides of the ring cavity are polarizing beam splitters, these ensure that the pump passes through the cell

and is not circulated through the cavity, the polarizing beam splitters also allow some of the generated laser field to escape the cavity allowing for measurement of the internal field. (See Figure 2).

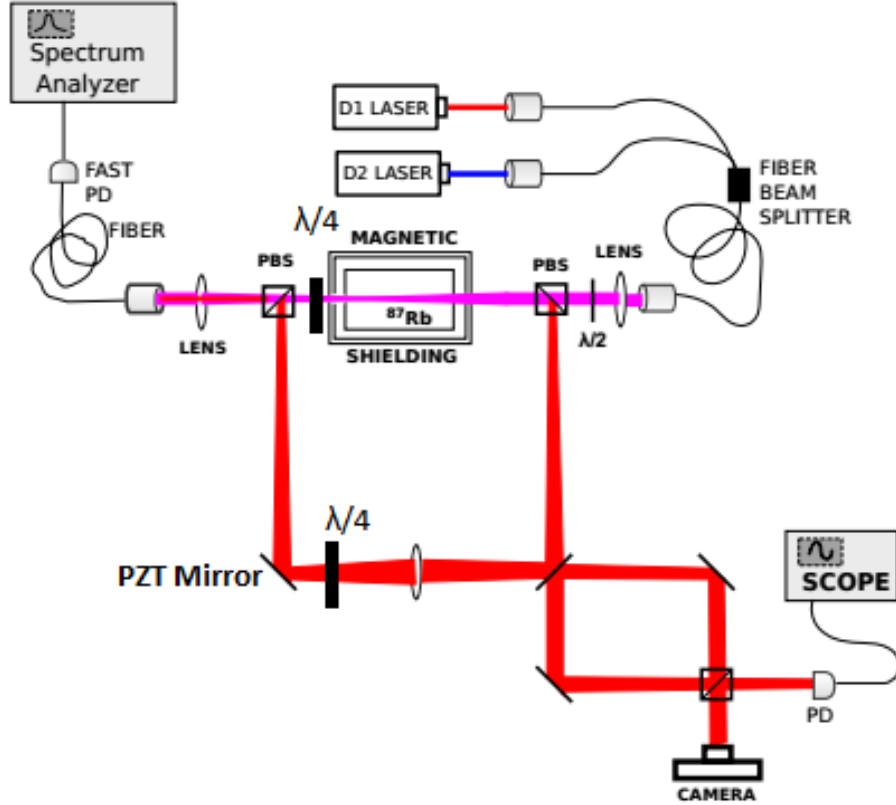


Figure 2: A simplified diagram of the cavity used in this experiment.

The degree to which the generated field is allowed to escape from the cavity is controlled by a quarter wave plate immediately after the rubidium cell. This quarter wave-plate controls the transmission of the cavity, and therefore controls the finesse of the cavity.

In order to provide sensitive measurements of sensitivity the length of the cavity must be modulated, either through rotation or through the elongation of the cavity. One of the mirrors on the table was mounted to a piezoelectric, this allows for electrical control of the cavity length.

Traditionally in a simple linearly configured cavity, concave mirrors are used to ensure cavity stability. The ring cavity configuration used in this experiment is not conducive to the use of concave mirrors, so

a lens placed inside of the cavity is used to maintain the stability of the system.

3.2 Measuring Gyroscope Sensitivity

An increase in gyroscope sensitivity would mean that smaller rotational velocities would register as a larger frequency splitting in the dispersive cavity than in the empty cavity. the magnitude of this increase in sensitivity can be characterized by a quantity henceforth referred to as pulling factor. Pulling factor is simply the ratio of the frequency splitting measured in the dispersive cavity to the frequency splitting expected in an empty cavity.

$$PullingFactor = \frac{\Delta f_{dispersive}}{\Delta f_{empty}} \quad (8)$$

based on the earlier theoretical considerations of the laser gyroscope, pulling factor should be equal to the inverse of the group index. In order to measure the sensitivity of the gyroscope, we measure the degree to which altering the length of the cavity effects the resonant frequencies of the cavity. Measuring the actual absolute frequency of the laser is extremely difficult. Instead of measuring the absolute frequency, heterodyne detection is used. Superimposing one of the pump fields with the generated gyroscope field a modulation of the optical power is induced hyperfine-splitting frequency of 6.83 GHz(see figure 3) this microwave frequency can be measured with relative ease using a fast photo-diode and a spectrum analyzer. Pulling factor is measured by the movement of beat note position while varying the length

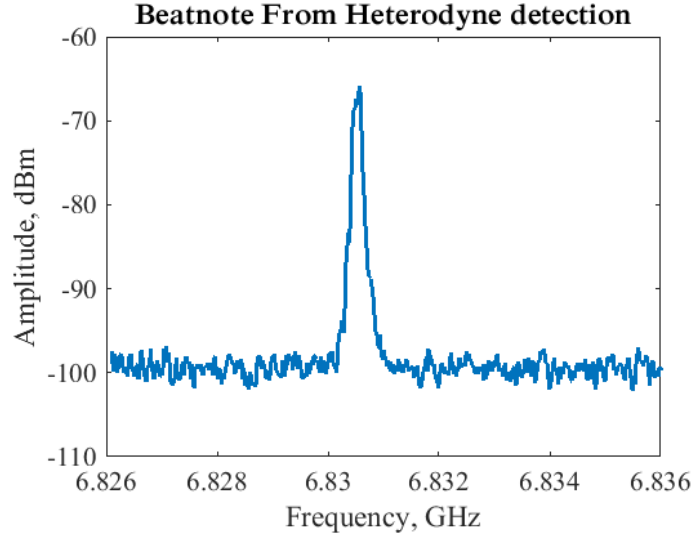


Figure 3: Typical Beat-note generated by mixing pump and lasing fields centered near 6.83 GHz.

of the cavity. Changes in cavity length can be readily converted into expected shifts in the resonant frequency of the empty cavity The slope of the resulting data is the ratio of the measured dispersive cavity to the expected shift in an empty cavity, in other words the slope is the pulling factor. (see figure

4)

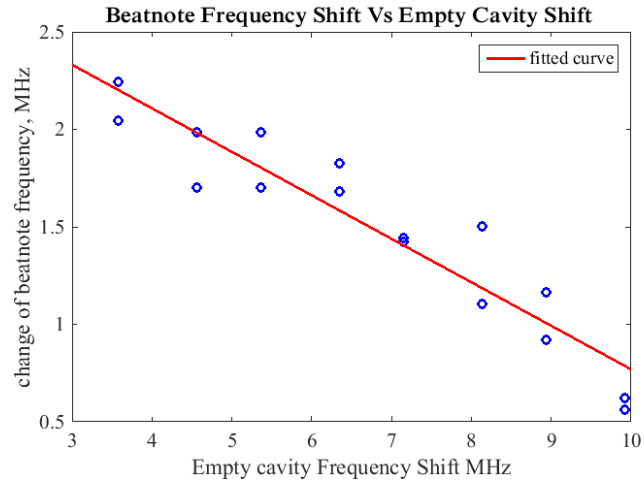


Figure 4: Beat-note measurements at different cavity lengths

Pulling factor is therefore extracted from data by fitting a line to the data and taking the slope.

3.3 Control Systems

In order to properly control the parameters necessary for the experiment a number of control systems had to be applied.

In order to carry out effective measurements of pulling factor careful control of cavity length is required. A proportional-Integral controller is used to stabilize the cavity length. Initially generation features from four wave mixing were used as lock signals, these signals tend to be very abrupt and tended to cease to be linear beyond a certain range. These properties limit the effectiveness of gyroscope modes as a lock signal. In order to stabilize the cavity length effectively a third field, far detuned from rubidium resonances. this field provides a sharp lock signal around the cavity resonances by slowly modifying the Detuning of the third field (referred to as the lock field) the length of the cavity can be stably controlled.

In order to fully explore the parameter space of the four wave mixing, careful control of the pump Detuning is required. In order to measure pump detuning a reference cell was used. The length of the reference cavity can be stably modulated using a high voltage signal. Calibration measurements of the reference cavity were made allowing for conversion between voltage changes, and length changes. By placing a cavity reference on top of a known atomic resonance and then altering the reference cavity length to move the reference cavity resonances to the desired Detuning allows for the calibrated Detuning of pump fields. This system can also be used in conjunction with a PID controller to stabilize pump Detuning.

3.4 Pump Power Control

Controlling the pump power in the experiment was first conducted by attenuating the pumps before injecting them through the rubidium cell. This method proved to place fairly stringent limitations on the experiment. The relatively weak external cavity diode lasers used as sources for the pump fields were further attenuated as they incurred losses when injected into optical fibers. The highest combined pump power achievable using this method were limited to a maximum around 10 mW. In order to increase the power of the system a semi-conductor amplifier system was added. The combined pumps are injected through a fiber into the laser amplifier and then through the gyroscope cell. The semiconductor amplifier increased the maximum pump power to the 200 mW range.

4 Results

The parameters theorized to have an effect on the properties of this four wave mixing scheme are numerous. Factors like Pump power, pump power distribution, rubidium vapor density, pump Detuning, and cavity finesse are hypothesized to have strong effects on pulling factor. The dependences are not necessarily independent of one another, altering one parameter may have drastic effects on the dependencies on other parameters. As a result dependences are measured in different regimes. As a target a pulling factor of greater than one would mean that the sensitivity of the gyroscope has exceeded the empty cavity regime.

4.1 Initial D1 Detuning Dependence

The first parameter dependence to be evaluated systematically was the D1 pump detuning. While measuring the initial D1 detuning dependence, the D2 pump was tuned to the peak generation at $F_g = 2 \rightarrow F_e = 1$ transition. The D2 pump had 2 mW of power inside the cell while the D1 pump had a power of 5 mW inside of the cell. The rubidium cell was at a temperature of 80 degrees Celsius. Measurements of pulling factor, and peak beat note amplitude were taken at various D1 detunings (see figures 5 and 6)

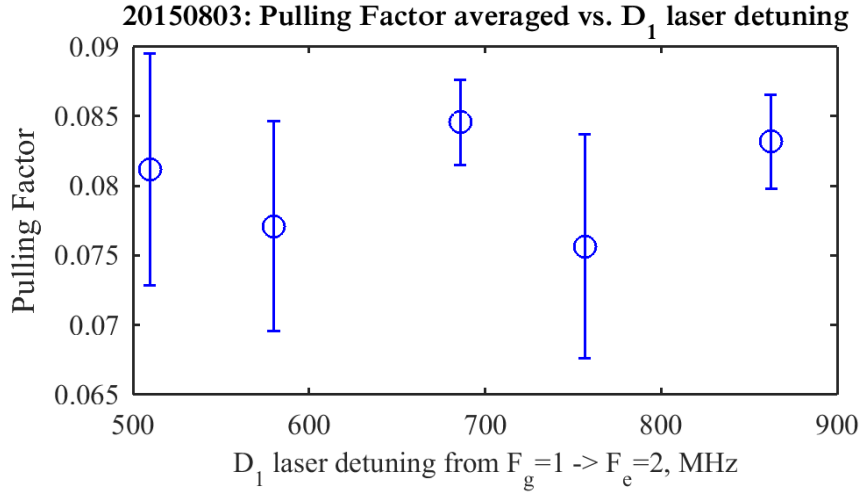


Figure 5: The initial Dependence of pulling factor on D1 pump detuning was not particularly strong. Pulling Factor in this regime pulling factor is limited to just under 0.1

In this regime the dependence of pulling factor on D1 Detuning appeared to be rather weak despite the detuning range of nearly 400 MHz pulling factors all are clustered fairly neatly between 0.075 and 0.085 with fairly broad error bars.

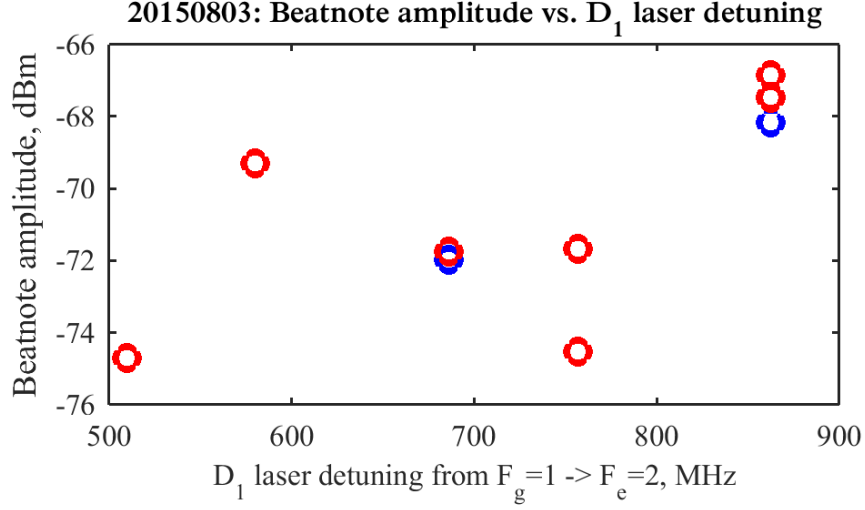


Figure 6: Similar to the Pulling factor dependence the initial dependence of beat-note amplitude on D1 pump detuning was not particularly strong.

4.2 Initial D2 Detuning Dependence

After measuring very little dependence of pulling factor on D1 Detuning under the previous conditions, the D2 pump detuning was the next parameter to be evaluated. For the duration of this particular experiment the D1 pump was tuned 314 MHz away from the $F_g = 1 \rightarrow F_e = 2$ transition. The D1 pump was injecting a 5mW of power into the rubidium cell, while the D2 pump was injecting around 2mW into the cell. The rubidium cell temperature was maintained at 80 Degrees Celsius. As in the previous experiment, measurements of beatnote amplitude were taken at various D2 detunings (See Figures 7 and 8)

In a contrast to the initial D1 pump detuning dependence the D2 pump detuning dependence was fairly strong. When the D2 pump was tuned to the $F_g = 2 \rightarrow F_e = 3$ transition pulling factor was double what it was when it was detuned 600MHz to the negative, and at least an order of magnitude larger than when detuned 75Mhz to the positive. Beat note amplitude began to roll off right around the area of strong pulling factor but dropped to its weakest point just before entering the region of low pulling factor.

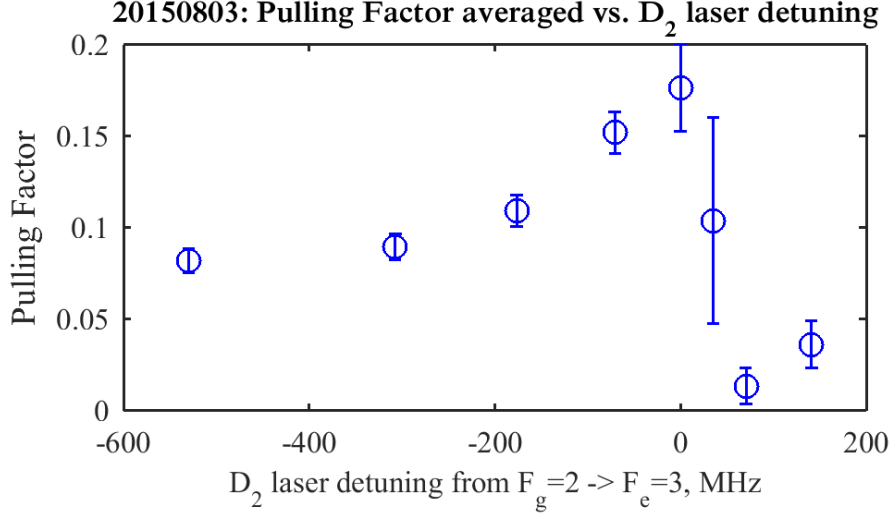


Figure 7: The initial dependence of pulling factor on D2 pump detuning is significantly stronger than the dependence on D1 detuning. A clear maximum is present near the $F_g = 1 \rightarrow F_e = 2$ resonance.

4.3 D1 Detuning Dependence under improved D2 Detuning Conditions.

After Maximizing the pulling factor with the D2 pump detuning under previous conditions. D1 dependence was re-evaluated under the new slightly improved conditions. for this experiment the D2 pump was tuned to the $F_g = 2 \rightarrow F_e = 3$ transition, where maximum pulling factor was observed in the previous experiment. The D2 pump was injecting 2 mW of power into the cell, and the D1 pump was injecting 5mW of power into the cell. the rubidium cell temperature was 80 degrees Celsius. Measurements of pulling factor, and peak beat note amplitude were taken at various D1 detunings (see figures 9 and 10)

With the new D2 pump Detuning the effect of D1 detuning became more apparent. Pulling factor was maximized when D1 was detuned 600 MHz from the $F_g = 1 \rightarrow F_e = 2$ Transition. In contrast to the D2 dependence, the D1 dependence did not display a sharp drop in pulling factor. Beatnote amplitude does not appear to be strongly correlated with pulling factor in this regime.

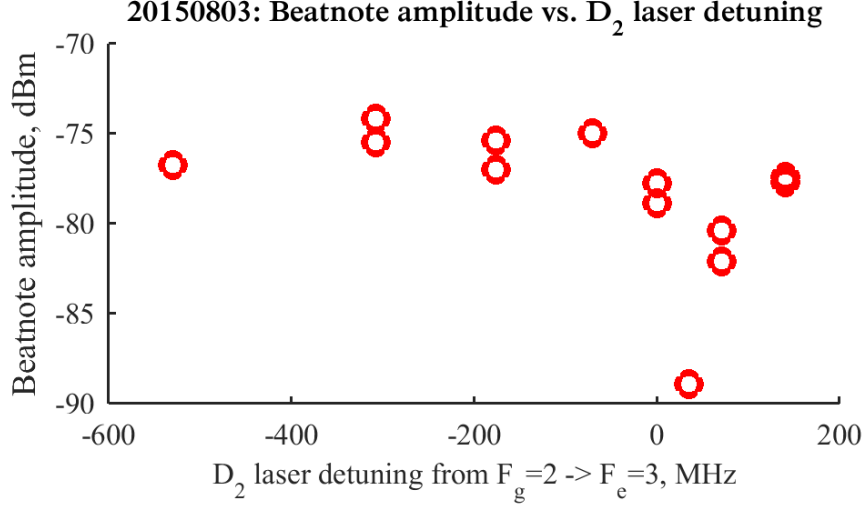


Figure 8: In this regime the beatnote amplitude drops strongly at small positive detunings from the $F_g = 1 \rightarrow F_e = 2$ resonance where maximum pulling factors were observed.

4.4 D2 Power Dependence (Low Power Regime)

Pump power is hypothesized to have a reasonably strong effect on pulling factor. In this configuration the easiest power parameter to adjust was D2 pump power. Power was altered by attenuating the D2 pump before it is injected into the rubidium cell. The D1 pump was tuned 314 MHz away from the $F_g = 1 \rightarrow F_e = 2$ transition while providing 5mW of power into the cell. the D2 pump was tuned to the $F_g = 2 \rightarrow F_e = 3$ transition, where maximum pulling factor was measured. Pulling factor and beat note amplitude were measured at various D2 pump powers. (see figures 11 and 12)

As the D2 pump power was increased the pulling factor continued to increase. There was no observed maximum in this dependence. It is unclear from the observed dependence whether increasing pump power would continue to cause drastic improvements to pulling factor or if increasing the power would just bring pulling factor closer to an asymptotic limit. At higher powers where higher pulling factor was observed the beat note amplitude was beginning to roll off.

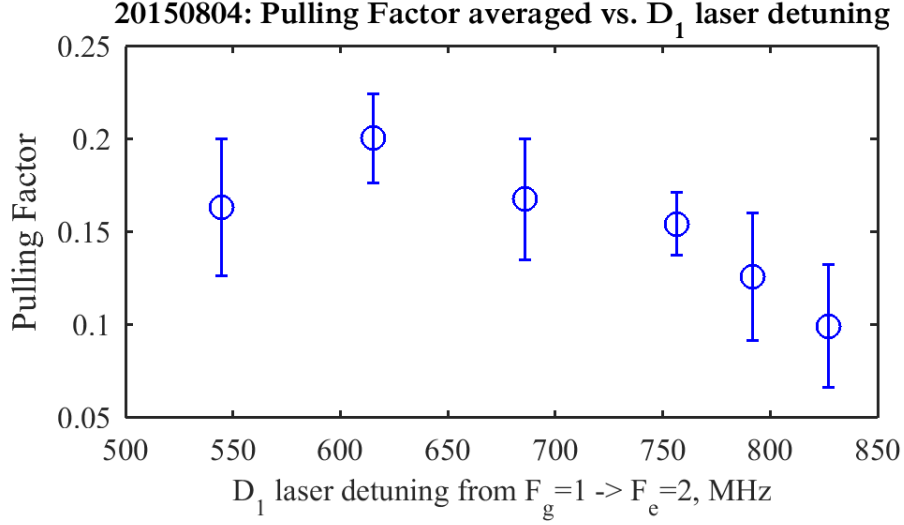


Figure 9: Under The improved D2 conditions the dependence of pulling factor on

4.5 Temperature Dependence

Temperature directly effects the density of the vapor in the ^{87}Rb vapor in the cell. The density of the ^{87}Rb vapor is theorized to have an effect on pulling factor. The temperature was controlled via a temperature controller that regulates the temperature of the rubidium cell. The pump fields were tuned to maximize generation. Pump powers were maintained at the levels of previous experiments, the D2 pump was injecting 2mW into the cell while the D1 pump was injecting 5 mW. Pulling factor and beat note amplitude where measured at various temperatures (see figure 13).

At these detunings and powers increasing the temperature caused fairly drastic drops in pulling factor. Increasing the temperature by 20 degrees the nearly dropped the pulling factor by nearly half. While decreasing pulling factor, increased temperatures caused fairly strong increases to beat note amplitude.

4.6 Finesse Dependence

Finesse is also theorized to have an effect on pulling factor. In this experiment some losses are required to measure the generated field, therefore higher finesse make measurements more difficult. However a high finesse is hypothesized to have strong contributions to pulling. To test the dependence, the pump fields were tuned to maximum generation and measurements were taken after varying the finesse (figure 14). Finesse was controlled by rotating a quarter wave plate that controls losses in the generated field inside the cavity.

The wave plate rotations conducted, represent a fairly wide range in finesse (nearly a 50 percent variation). Contrary to expectations pulling factor was lowest at the point of highest finesse (polarizer

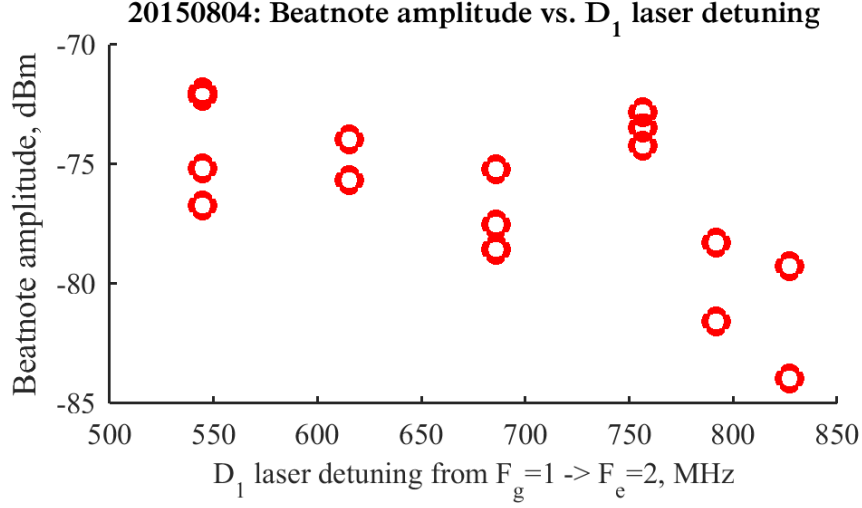


Figure 10: Final dependence of beat-note amplitude on D1 pump detuning

position: 203 Degrees).

4.7 D2 detuning Dependence (High Power Regime)

The addition of a semiconductor laser amplifier to system allows for pump powers an order of magnitude higher than in the previous experiments. The first dependence measured with the inclusion of the Semiconductor amplifier was the dependance on D2 pump detuning. The D1 pump was detuned +314 MHz away from the $f_g = 1 \rightarrow F_e = 2$ resonance. The total pump power injected into the cell was 181mw with a distribution of 58% of the power coming from D1 and 42 % coming from D2. Measurments of pulling factor were made at various D2 pump detunings.

In the high power regime Pulling factors are significantly higher than they were at low powers. In the low power regime (total pump power 10 mW) pulling factors were limited to around 0.3. Increasing the power allowed for drastic improvements to pulling factor as the minimum pulling factor observed in this experiment was around 0.6 nearly double the maximum pulling factor observed in the low power experiments. The pulling factor approached and may have exceeded the limiting pulling factor of one. In addition to the strong improvement to pulling factor the increased pump power provided for significantly stronger beatnote signals.

5 Discussion

The properties of the optical gyroscope cavity under consideration in this experiment are sensitive to a wide range of factors.

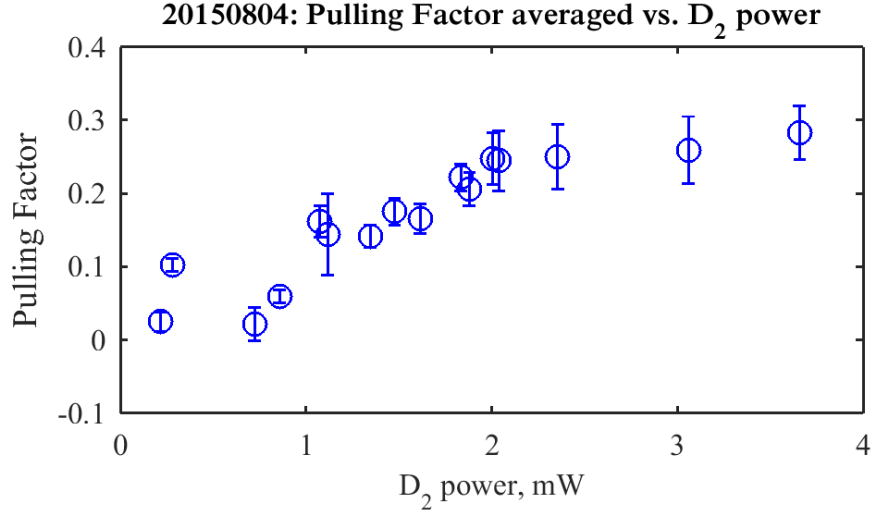


Figure 11: The dependence of pulling factor on D2 pump power showed consistently improved pulling factors at higher powers.

The most obvious dependence was the strong dependence of pulling factor on power. Across the board it appears that increasing pump power increases the pulling factor. seemingly just by increasing the pump power pulling factors can be brought from near 0 to approaching unity. In addition to the improvement of pulling factor the higher pump powers correspond to higher powers of the field generated within the cavity, the high power of the gyro field corresponds to a higher beatnote amplitude and therefore detection of the beatnote is easier. Increased power seems to be the easiest parameter to modify to achieve large pulling factors.

Although higher pump powers yield to both higher pulling factor and a higher beatnote amplitude it appears that the connection between the two two properties of the gyro signal are not necessarily correlated. Increasing the Density of the rubidium vapor caused the pulling factor to drop while increasing the beatnote amplitude.

Detuning dependences seem to be more strongly Dependant on the other parameters in the system. the Pump Detuning dependences were altered by factors such as the power of the pumps and the Detuning of the other pump.

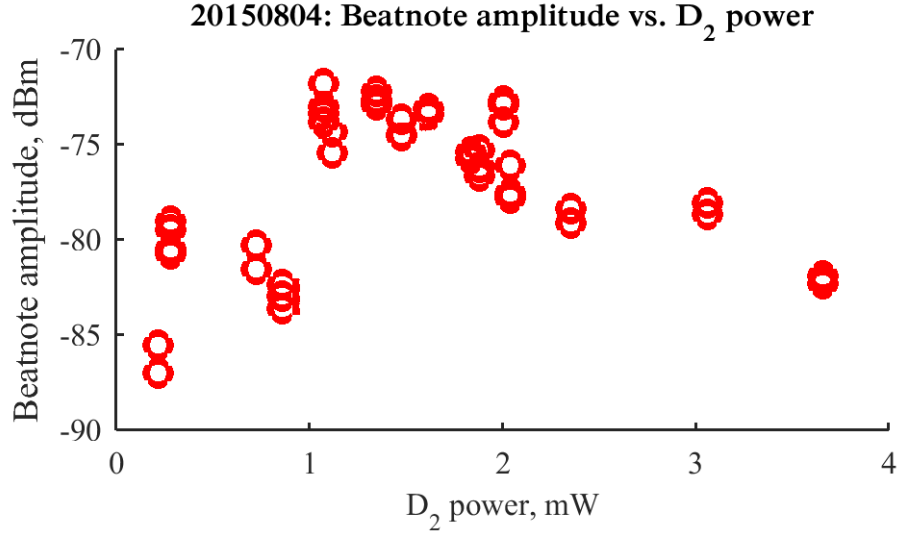


Figure 12: At higher D_2 powers in this regime the beat note amplitude began to roll off.

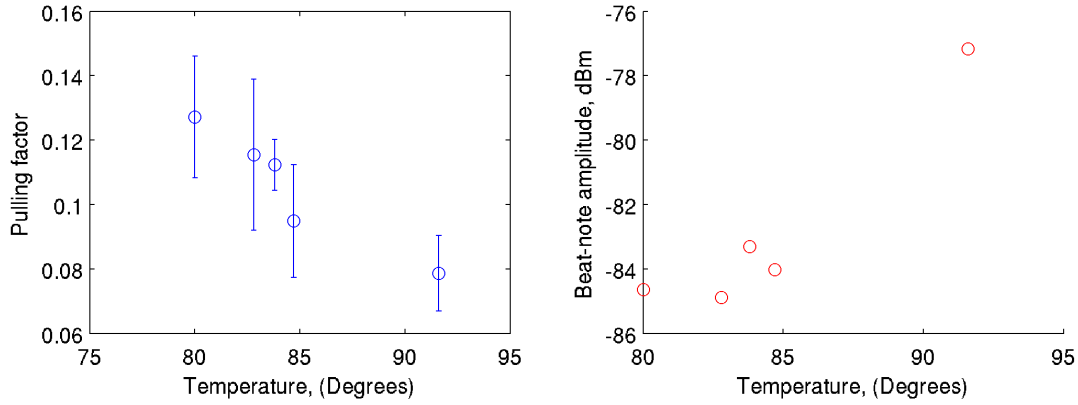


Figure 13: Dependence of pulling factor on Temperature

6 Conclusions

Tuning various parameters can have very drastic effects on the sensitivity of this gyroscope cavity. Most notably higher power regimes tend to provide improved sensitivity. Further improvements can make the cavity very sensitive. Despite low initial pulling factor observations of around 0.1, tuning parameters especially power and pump detuning can drive pulling factors to approach unity. Further exploration of the parameter space may lead to pulling factors reliably greater than unity.

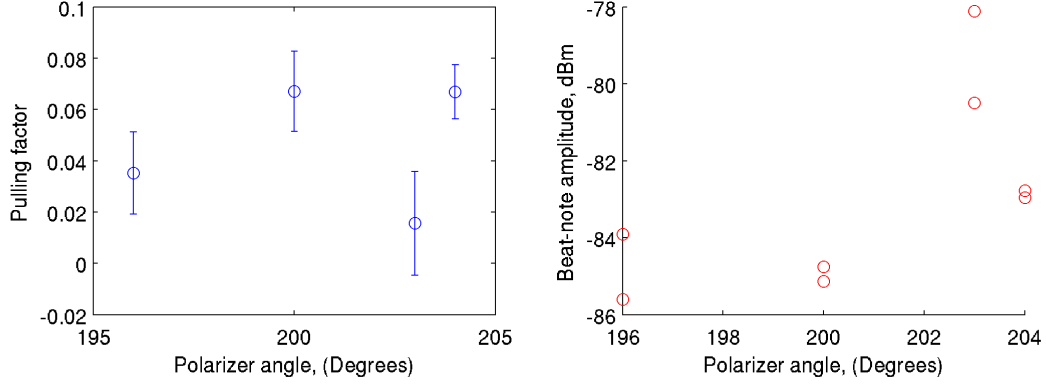


Figure 14: Dependence of pulling factor on quarter wave plate position.

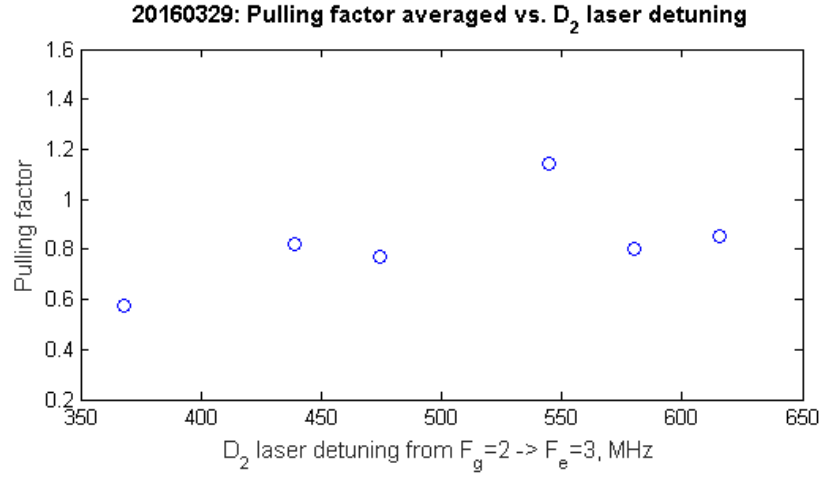


Figure 15: Pulling factor dependence on D2 Detuning in the high power regime

7 Acknowledgments

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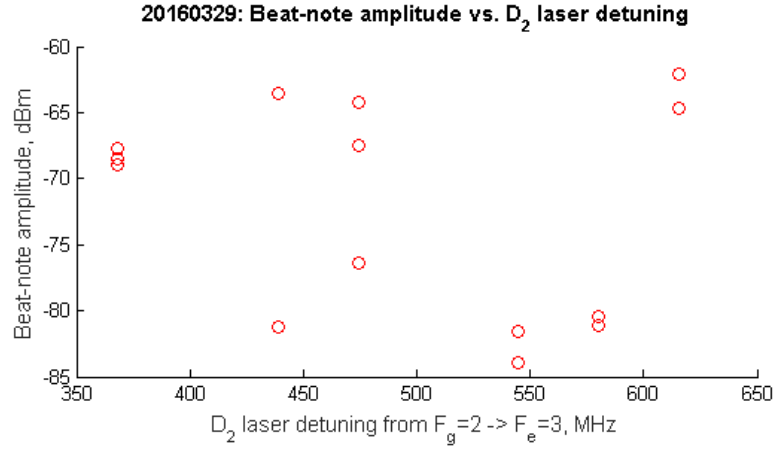


Figure 16: Final dependence beat-note amplitude on D2 pump power

References

- [1] The Editors of Encyclopdia Britannica. Inertial guidance system. *Encyclopdia Briannica*, 2015.
- [2] Adi Shamir. An overview of optical gyroscopes theory, practical aspects, applications and future trends. 2006.