Optical Resonators With Whispering-Gallery Modes—Part I: Basics

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(Invited Paper)

***Abstract*—We briefly review basic properties of dielectric whispering gallery mode resonators that are important for applications of the resonators in optics and photonics.**

***Index Terms*—Four-wave mixing, high-order optical filters, lasers, laser resonators, monolithic optical total internal reflection resonators, morphology dependent resonances, nonlinear optics, optical filters, optical resonators, parametric optics, Q-factor, solid-state lasers, spectroscopy, tunable filters, wave mixing, whispering-gallery mode (WGM) resonators.**

I. Introduction

CONVENTIONAL optical resonators consisting of two or more mirrors are utilized in all branches of modern linear and nonlinear optics, where multiple recirculation of optical power is required to maintain laser oscillation, to increase the effective path length in spectroscopic or resolution in interferometric measurements, to enhance wave mixing interactions, etc. Crucial properties of optical resonators, such as high quality-factor and finesse, can be achieved when the highest reflectivity and low loss mirrors, as well as the most transparent optical elements are used for their construction.

Despite their versatility, traditional Fabry–Pérot (FP) resonators and their folded or ring varieties have remained fairly complex and expensive devices, large in size, difficult in assembly, and prone to vibration instabilities because of relatively low-frequency mechanical resonances. For certain applications, stability and small modal volume are of great importance, however miniaturization of conventional FP resonators is either very complicated (when high-finesse mirrors are utilized), or yields rather low quality (Q)-factors.

Search for high quality monolithic resonators for lasers has produced elegant solutions such as distributed feedback resonators or fiber resonators with Bragg reflector mirrors. In both cases the resonators represent linear structures in which resonances are produced by coupling and interference of counter propagating waves.

We focus in this review on rapidly growing field of monolithic resonators in which the closed trajectories of light are supported by any variety of total internal reflection in curved and polygonal transparent dielectric structures. Following the traditional term of microwave electronics, we call these resonators "open dielectric resonators." The circular optical modes in such resonators, frequently dubbed as whispering-gallery modes (WGMs), can be understood as closed circular beams supported by total internal reflections from boundaries of the resonators. Extremely high values of Q-factor can be achieved in WGMs of very small volume, in certain cases as small as cubic wavelength, with appropriately designed high precision dielectric interface, and with use of highly transparent materials.

The simplest geometry of such resonators is either a ring, or a cylinder, or a sphere. When the reflecting boundary has high index contrast, and radius of curvature exceeds several wavelengths, the radiative losses, similar to bending losses of a waveguide, become very small, and the Q becomes limited only by material attenuation and scattering caused by geometrical imperfections (e.g., surface roughness).

Fabrication of the open dielectric resonators can be very simple and inexpensive, and they lend themselves very well to either hybrid or on-chip integration. Unique combination of very high (Q as high as 10^{10}) and very small volume has attracted interest to applications of the resonators in numerous fundamental science and engineering applications. Small size also results in excellent mechanical stability and easy control of the resonator parameters. The resonators can be easily tuned, stabilized, and integrated into the optical networks.

In this paper we review properties of whispering gallery resonators (WGR) especially important for their applications. We do not focus on exact solutions of the wave equation for the resonators, but rather try to cover practical aspects as well as recent progress related to problems of external coupling to the resonators, materials suitable for the resonator fabrication, and simple methods of estimation of WGRs properties. The detailed as well as exact description of properties of the resonators can be found in textbooks [1], [2] and reviews [3]–[8]. The followup paper of this special issue [9] is devoured to review of the applications of the WGRs.

II. Early Studies of WGMs

The history of WGMs as "WGMs" was started almost a century ago by work of Lord Rayleigh, who studied propagation of sound over a curved gallery surface [10]–[12]. On the other hand, a little bit earlier, in 1909, Debye derived equations for the resonant eigenfrequencies of free dielectric and metallic spheres, which naturally take into account WGMs [13]. Those equations can also be deducted from the theoretical studies by

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Mie on the scattering of plane electromagnetic waves by spheres, published in 1908 [14]. A possibility of existence of modes with very high Q-factors in axisymmetrically shaped open dielectric resonators was pointed out in [15]. Generalized properties of electromagnetic resonances in dielectric spheres, including their Q-factors, was widely discussed afterwards, with emphasis on the microwave, not optical, modes of the spheres (see, e.g., [16], [17]), though, naturally, the equations are the same as in optics.

The first observations of WGMs in optics can be attributed to solid state WGM lasers. Laser action was studied in Sm:CaF_2 crystalline resonators [18]. The size of the resonators was in the millimeter range. Microsecond-long transient laser operation has been observed in [19] with a several millimeter ruby ring at room temperature. Transient oscillations were attributed to pulsed laser excitation of WGMs with Qs of 10^8–10^9.

WGMs were first observed by elastic light scattering from spherical dielectric particles in liquid resonators [20], [21]. It was recognized that WGMs could help in measurements of spherical particle size, shape, refractive index, and temperature [22], [23]. WGMs were used to determine diameter of optical fiber [24]. Strong influence of WGMs on fluorescence and Raman scattering was recognized in [25]–[27], and [28]–[30], respectively. Laser action in a free droplet was first studied in [31], [32].

III. Near-Field Coupling to High-Q Optical WGMs as the Turning Point in the Technology Development

The problem of efficient and robust coupling to WGMs is, in our opinion, the crucial problem for practical applications of WGRs. It is due to the successful development of waveguide coupling technique, the first commercial WGR-based devices have became available [33].

Required operational principles of the coupling devices are similar and based on 1) phase synchronism, 2) optimal overlap of the selected WGM and the coupler mode, 3) selectivity, and 4) criticality. Only phase-matched evanescent field couplers posses those properties.

A. Free-Beam Coupling

Theoretically, if the main loss mechanism in the resonator is the radiative loss due to curvature (i.e., when material attenuation and scattering can be neglected), one can couple light into a WGM if it comes in the exact shape that coincides with the emission pattern of leaky WGMs. The problem is that this pattern is an exotic helical wave with large orbital momentum which expands into a spherical wave.

Coupling efficiency η of continuous wave light into a ring resonator could be quantified as

$$\eta = \frac{\pi}{\mathcal{F}}\frac{P_{\rm in}}{P} \qquad (1)$$

where $P_{\rm in}$ is the value of the power circulating inside the resonator, P is the value of external pump power, and $\mathcal{F}$ is the finesse of the loaded resonator. The resonator is undercoupled if $\eta < 1$, critically coupled if $\eta = 1$, and overcoupled if $2 > \eta > 1$.

Free beams do not efficiently couple to ultrahigh Q WGMs in large enough resonators ($\eta \ll 1$). Indeed, such a coupling must rely upon radiative exchange of a WGM with external space. This radiative exchange is very strongly dependent on the resonator radius and becomes extremely small when the radius exceeds several wavelengths. Radiative coupling of a microsphere and a beam of light is described in generalized Lorentz–Mie scattering theory [34]–[37], or modified ray theory described in [38] and confirmed in [39].

Let us roughly estimate the efficiency of coupling of a pumping light beam interacting with an ideal sphere. The beam cross section radius is comparable with the sphere radius a (total cross section $\sim 2\pi a^2$). Scattering cross sectional mode area is $\sim a\lambda$ [38]. Therefore, only $\sim \lambda/(2\pi a)$ part of the total power interacts with the sphere.

To estimate the ratio of the mode coupling quality factor $Q_{\rm c}$ and total quality factor Q we assume that the radiative losses and optical pumping are of the same origin and are proportional to the interaction surface area [40]. The radiative emission occurs from the entire sphere surface (area equal to $4\pi a^2$). The optical pumping is going through a surface belt with thickness $\sim \lambda$ (area equal to $2\pi a\lambda$). Therefore, the ratio of the coupling and total quality factors is proportional to the sphere radius $Q_{\rm c}/Q \sim a/\lambda$.

The coupling efficiency depends on both, the ratio between the amount of radiative power of the pump beam penetrating into the mode scattering area and the total optical pump power, and the ratio of the coupling and total quality factors. Using the estimations we conclude that the coupling efficiency is less than $\eta < (\lambda/a)^2$.

In reality, the radiative coupling to a WGR is much less than estimated above. Radiative coupling is negligible in vast majority of experiments and applications with resonators exceeding few tens of micron in diameter. For example, direct calculation of the radiative Q-factor yields an enormous value 10^{73} at $\lambda = 600$ nm for a water droplet with radius 50 μm [6]. This value is more than 60 orders of magnitude higher than the limit imposed on Q by light scattering and absorption in best transparent materials. This explains additional restriction on direct radiative coupling: on top of the above cross section limitation, efficiency of such coupling will be further reduced by the ratio $Q_{\rm att}/Q_{\rm rad}$, turning it virtually into zero.

It should be mentioned here that nonnegligible free beam coupling can still be used for certain experiments with WGM of rather high Q, $10^5 - 10^8$, if it is limited by sizable contribution of *scattering* [41], [42], [44]–[47] in overall loss budget. This type of coupling was used in resonator-enhanced nonlinear optics experiments with aerosol droplets.

B. Critical Coupling

Efficient controllable coupling, the critical requirement of feasibility of practical applications of the resonators, can only be achieved via the near field of a WGM. In this case, coupling is based on phase- and mode-matched power exchange between the WGM and guided wave in a specially engineered coupler (a waveguide or a prism). Parasitic coupling to unwanted modes is quantified by the "ideality"—the ratio of power coupled to a desired mode divided by power coupled or lost to all modes (including the desired mode).

For efficient energy exchange, resonator should be, not only ideally, but also critically coupled to a chosen waveguide or fiber. The notion of critical coupling, fundamental in RF engineering (see, e.g., [48], [49]), has been recently "rediscovered" for optical WGM resonators [50]. Indeed, unlike lossless FP resonators whose fixed loss ($1 - R$, where R is mirror reflectivity) is identical to its coupling loss, the strength of evanescent coupling to WGM resonators is completely independent of their intrinsic loss. Criticality implies that to provide 100% energy exchange at resonance, coupling strength between waveguide and resonator must match the intrinsic loss (i.e., its coupling to the heat bath).

C. Evanescent Field Couplers

Prism couplers with frustrated total internal reflection are among the oldest methods to couple light and WGMs. The simplest yet efficient prism technique is based on three main principles. First, the input beam is focused inside the high-index coupling prism under the angle that provides phase matching between the evanescent wave of the total internal reflection spot, and the WGM, respectively. Second, the beam shape is tailored to maximize the modal overlap in the near field. And third, the gap between resonator and prism is optimized to achieve critical coupling [51]–[53]. Prism-waveguide coupling efficiency with more than 90% was demonstrated in [54], [55]. Prism coupling to WGMs was investigated in [42], [43], [56]–[59]. The best efficiency of prism coupling to WGMs reported up to date is $\sim$80% [43]. Achieving high ideality with prism coupler is complicated because of continuous nature of the modal space in the prism (modes are free beams).

At present, in addition to the well-known prism coupler, coupler devices include side-polished fiber couplers [60]–[62] (having limited efficiency owing to residual phase mismatch), fiber tapers [63]–[70] (almost $\sim$100% coupling achieved), hollow fibers [71], "pigtailing" technique with an angle-polished fiber tip in which the core-guided wave undergoes total internal reflection [72], [73], and special technique of coupling of the cavities and semiconductor lasers [74], [75].

Planar waveguides are used to couple to ring and disk WGRs [76], [77]. Strip-line pedestal antiresonant reflecting waveguides are for robust coupling to microsphere resonators and for microphotonic circuits [78], [79]. An analytical method for the calculation of the coupling between a microdisk WGR and a straight waveguide was presented in [64], [80].

The most efficient coupling to date was realized with tapered fiber couplers. Fiber taper—a single-mode bare cylindrical waveguide of optimized diameter (typically few micron for near infrared wavelength) can be placed along the resonator perimeter allowing simple focusing and alignment of the input beam, as well as collection of the output beam. In the context of critical coupling, it filters all other waveguide modes very efficiently, save the fundamental, at both the input and the output. The mode filtering performed by the fiber taper before and after the coupling region results in a taper-resonator system which is, for all intents and purposes, a single-mode coupler and thus effectively an ideally matched coupler. Efficiency of tapered fiber couplers reaches 99.99% for coupling fused silica resonators [68]. Similar couplers were used in photonic crystal resonators as well [69], [70]. Unfortunately, fiber tapers as unclad, unsupported waveguides, are very fragile and only applicable to resonators with refractive index similar to silica (1.4–1.45). They cannot be used with higher index glass and crystalline WGRs that have attracted increasing attention in the last few years.

D. Symmetry Breaking Techniques

Coupling with comparably low Q, low-azimuthal order WGMs can be realized using symmetry breaking techniques. For example, a directional coupling of light output from WGM microdisk lasers was described in [81]. Patterned asymmetries in the shape of microdisk resonators provide control of both direction and intensity of light output without dramatically increasing laser thresholds. A "microgear" laser was studied in [82]. Directional emission from asymmetric as well as elliptical WGRs was discussed in [83]–[87]. Directional coupling via linear defect at some distance away from the circumference of a microdisk resonator was studied in [88].

IV. Transparent Materials for WGM

A. Liquids and Amorphous Materials

Low optical attenuation in the resonator material is a fundamental prerequisite for obtaining high values of quality factor. Since many simple organic and nonorganic liquids are highly transparent, it is not surprising that high-Q WGMs were observed indirectly in numerous experiments with aerosol droplets which represent nearly ideal spheres shaped by surface tension forces. Micron-sized liquid droplets were widely used for cavity enhanced spectroscopy (see [89] for review). They were used in studies of fluorescence and lasing of dye containing liquids [31], [90]–[92]. Stimulated Raman scattering was studied in CS_2 [93], CCl_4 [94], water [95], glycerol [96], and other droplets.

Applications of WGMs in liquid resonators are limited because of the difficulty in manipulation, short evaporative lifetime, and mechanical instabilities. Only solid state WGM resonators can become a basis of practical photonic devices. The first ultrahigh-Q solid WGM microresonators were nothing else than solidified droplets of molten silica [97]. Amorphous materials, like fused silica, can possess very small optical attenuation. In the "blue region," from ultraviolet (UV) to approximately 1.5 μm wavelength, the highest quality factor of WGMs remains limited by Rayleigh scattering on molecular scale volume density fluctuations and residual surface roughness [47]. Any additional roughness would result in additional limitation of the Q. In the "red region," from 1.5 μm further into the infrared area of spectrum, the Q of silica resonator starts to be limited by lattice absorption ("multiphonon edge"). Near the middle of transparency window (at 1.5 μm) the Q of fused silica resonators is affected by chemosorption of OH^- ions and water [45], [46].

Another example of amorphous resonators are polymer spheres [98]–[102]. Though fabrication of polymer resonators is simple enough, their performance is limited because of strong absorption of light in the polymers. The Q values of polymer microresonators is limited by Rayleigh scattering, band-edge absorption, and vibrational absorption in the visible spectral range.

B. Crystals

Single crystals have not been extensively used in recent WGM studies despite of both their useful linear and nonlinear properties, attractive for photonics applications, and very small attenuation in the middle of the transparency window. The interest in crystalline WGRs has returned very recently with the demonstration of electro-optic crystalline resonators [103]–[106], being incremental for several decades, since the early studies of crystalline resonators [18], [19].

It is expected that the crystals would have less loss than fused silica because crystals theoretically have a perfect lattice without imperfections, inclusions, and inhomogeneities, which are always present in amorphous materials. The window of transparency for many crystalline materials is much wider than that of fused silica. Therefore, with sufficiently high purity material, much smaller attenuation in the middle of transparency window can be expected—as both Rayleigh scattering edge and multiphonon absorption edge are pushed further apart towards ultraviolet and infrared regions, respectively. In addition, crystals may suffer less, or not at all, from the extrinsic absorption effects caused by chemosorption of water.

For example, absorption of sapphire determined by light scattering due to imperfection of the crystalline structure is less than $\alpha = 1.3 \times 10^{-5}$ cm^{-1} at $\lambda = 1$ μm [108], which corresponds to $Q \simeq 8 \times 10^9 (Q = 2\pi\sqrt{\epsilon_0}/(\lambda\alpha), \sqrt{\epsilon_0} = 1.75)$. Light absorption for the crystalline quartz should be better than absorption in fused silica, which is extensively studied for fiber optic applications, with $\alpha \leq 5 \times 10^{-6}$ cm^{-1} at $\lambda = 1.55$ μm [107]; this corresponds to $Q \geq 1.2 \times 10^{10}$.

Melting is obviously not suitable for materials with crystalline structure, because it destroys the initial crystal purity and stoichiometry. Moreover, during solidification, the original spherical droplet of the melt turns into a rough body with multiple facets and crystal growth steps. Surprisingly, adaptation of optical polishing techniques, previously used for planar or axial optics, for fabrication of polished spheroidal perimeter, has yielded millimeter-diameter crystalline WGRs with very high Q-factors, up to $(Q = 2 \times 10^{10})$ [104], even higher than that of surface-tension-formed resonators [45].

Original crystal structure and composition is preserved in the polished resonators, and the unique linear and nonlinear crystal properties are enhanced with the small volume of the high-Q cavity. Total internal reflection at the walls of the WGRs provides the effect of an ultrabroad-band mirror, allowing very high Q-factors across the whole material transparency range. This property makes crystalline WGRs a unique tool for optical materials studies.

There is very little consistent experimental data on small optical attenuation within transparency windows of optical crystals. For example, the high sensitivity measurement of the minimum absorption of specially prepared fused silica becomes possible only because of kilometers of optical fibers fabricated from the material. Unfortunately, this method is not applicable to crystalline materials. Strictly speaking, fibers have also been grown out of crystals such as sapphire [109], but attenuation in those (few decibel per meter) was strictly determined by scattering of their surface. Therefore, other methods of analysis of optical absorption in crystals should be used. The most sensitive calorimetry methods for measurement of light absorption in transparent dielectrics give an error on the order of $\Delta\alpha \geq 10^{-7}$ cm^{-1} [110]. Several transparent materials have been tested for their residual absorption with calorimetric methods, while others have been characterized by direct scattering experiments [108], [111], [112], both yielding values at the level of few ppm/cm of linear attenuation, which corresponds to the Q limitation at the level of 10^{10}. The question remains if this is a fundamental limit, or the measurement results are limited by residual doping or non-stoichiometry of the crystals used.

WGRs have already helped in studies of the crystal properties. For example, WGRs made of $LiNbO_3$ were recently used to demonstrate resonant interaction of light and microwaves [105], [106], [113]–[117]. The maximum quality factor of WGMs in the resonators reported in [117] was less than 5×10^6 at $\lambda = 1.55$ μm, which approximately corresponded to values expected from the intrinsic absorption of congruent lithium niobate cited by crystal producers ($\alpha \leq 5 \times 10^{-3}$ cm^{-1}). This value was corrected in experiments with different samples of the crystals and improved polishing technique, 2×10^8 at $\lambda = 1.55$ μm [104]. On the other hand, $Q \approx 2 \times 10^8$ at $\lambda = 2.014$ μm ($\alpha \leq 5 \times 10^{-4}$ cm^{-1}) was reported earlier for a multiple total-internal-reflection resonator [118]. The same value of $Q = 2 \times 10^8$ at $\lambda = 1.3$ μm was obtained in [104]. This suggests that the Q-factors achieved were either limited by the fabrication process, or that the values for the absorption of congruent lithium niobate listed in the literature are inaccurate.

Selection of material for the highest Q WGRs must be based on fundamental factors such as the widest transparency window, high purity grade, and environmental stability. For this reason, alkali halides that have the widest transparency, as known in spectroscopy, have to be rejected based on their hygroscopic property and sensitivity to atmospheric humidity. Bulk losses in continuous solid transparent materials can be approximated with the phenomenological dependence [111]

$$\alpha \simeq \alpha_{\rm UV}\, e^{\lambda_{\rm UV}/\lambda} + \alpha_{\rm R}\lambda^{-4} + \alpha_{\rm IR}e^{-\lambda_{\rm IR}/\lambda} \qquad (2)$$

where $\alpha_{\rm UV}$, $\alpha_{\rm R}$, and $\alpha_{\rm IR}$ describe the blue wing (primary electronic), Rayleigh, and red wing (multiphonon) losses of the light, respectively; $\lambda_{\rm UV}$ and $\lambda_{\rm IR}$ stand for the edges of the material transparency window. This expression does not take into account resonant absorption due to possible crystal impurities. Unfortunately, even phenomenological coefficients in (2) are not always known.

One of the most interesting candidates for fabrication of high-Q WGM resonators is calcium fluoride. It has attracted a lot of attention because of its use in ultraviolet lithography applications at 193 and 157 nm. Very large ultrapure uniform crystals of this material have been grown suitable for wide aperture optics and are commercially available. According to the recently reported measurements on scattering in CaF_2

(yielding the value of $\alpha = 3 \times 10^{-5}$ cm^{-1} at 193 nm [112]) extremely small scattering can be projected in the near infrared band, suggesting the feasibility of Q at the level of 10^{13}, while the achieved Q was 2×10^{10} [104].

It is obvious that the Q-factor of a WGR can be influenced by the resonator support. The support structure should be attached out of the WGM localization to reduce the influence. This is not always possible in the case of packaged resonators, and then the material properties not only of the resonator, but also of the support become important. The effects of the substrate and the support pedestal on microdisk performance was studied numerically using the finite-difference time-domain method [119]. The field distributions, resonant frequencies, and Q-factors of the WGR were computed at various distances from the substrate and for different pedestal sizes. The results showed that the supporting structure can significantly distort the field distribution in the disk, and therefore have effects on both resonant frequency and Q-factor, which are major parameters in the design of the WGR.

V. Summary of Linear Properties of WGMs

A. *Spectral Properties of WGMs in a Sphere*

Spectra of WGRs are determined by their shape and/or by spatial distribution of the refractive index inside the resonator. A general analytic solution to describe those spectra is nonexistent. However, the modal structure and spectra of WGMs in various axial symmetry geometries has been studied using numerical and approximate analytical methods [120], [121]. The simplest eigenvalue and eigenfrequency problem for electromagnetic field propagation in a dielectric sphere and cylinder has been solved exactly in, e.g., [1]. Theory of WGM resonances in spherical WGR is developed in [122]–[124] in analogy with quantum-mechanical shape resonances. Orthogonality of WGM was proven in [125]. An analytical expression for the spectrum of the high-order WGMs is derived in [126], [127].

Let us present below a simple method for approximate description of high-order WGMs [56], [128]. We consider a dielectric sphere with dielectric constant distribution $\epsilon(r)$ which depends on the radius r only. The electric field in the sphere obeys the Maxwell

$$\nabla \times (\nabla \times \mathbf{E}) + \frac{\epsilon(r)}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad (3)$$

where c is the speed of light in the vacuum. Presenting the electric field as $\mathbf{E} = \int_0^\infty d\omega \mathbf{e}(r) \exp(-i\omega t)$, we rewrite the above equation as

$$\nabla \times (\nabla \times \mathbf{e}) - k^2 \epsilon(r)\mathbf{e} = 0 \quad (4)$$

where $k = \omega/c$ is the wave vector. (4) may be solved in terms of TE and TM modes. Keeping in mind that $\nabla \cdot (\epsilon \mathbf{e}) = 0$ we write

$$\mathbf{e} = \sum_{\nu,m} \frac{1}{r}\left[\Psi \mathbf{Y}_{\nu,m} + \frac{1}{\epsilon(r)} \nabla \times (\Phi \mathbf{Y}_{\nu,m})\right] \quad (5)$$

where the radial functions $\Psi(r)$ and $\Phi(r)$ stand for TE and TM modes respectively, $\mathbf{Y}_{\nu,m}$ are vector spherical functions with angular number ν and magnetic number m. It is worth noting that modes of an infinite dielectric cylinder may be described in a similar way.

Radial field distribution for TE modes, for instance, of a dielectric cavity (sphere or cylinder) can be described by

$$\frac{\partial^2 \Psi}{\partial r^2} + \left[k^2 \epsilon(r) - \frac{\nu(\nu+1)}{r^2}\right]\Psi = 0 \quad (6)$$

where ν is angular momentum number ($\nu = 0, 1, 2, 3, \ldots$ for a sphere, $\nu = 1/2, 3/2, 5/2, \ldots$ for an infinite cylinder). Electric field distribution has a dependence $\Psi(r)/r$ for a sphere and $\Psi(r)/\sqrt{r}$ for a cylinder.

Equation (6) has an exact solution for a homogeneous dielectric spherical/cylindrical cavity with $\epsilon(r) = \epsilon_0 = \text{const}$. This solution reads $\Psi(r) = J_{\nu+1/2}(kr)$, where $J_{\nu+1/2}(kr)$ is the Bessel function of the first kind. The mode spectrum is determined by the boundary conditions $\Psi(r) \to 0$ for $r \to \infty$ and 0. For the case of high TE mode order $\nu \gg 1$

$$k_{\nu,q} \simeq \frac{1}{a\sqrt{\epsilon_0}}\left[\nu + \alpha_q\left(\frac{\nu}{2}\right)^{1/3} - \sqrt{\frac{\epsilon_0}{\epsilon_0 - 1}} + \frac{3\alpha_q^2}{20}\left(\frac{2}{\nu}\right)^{1/3} + O(\nu^{-2/3})\right] \quad (7)$$

where a_q is the qth root of the Airy function, a is the radius of the resonator, $Ai(-z)$ [129], which is equal to 2.338, 4.088, and 5.521 for $q = 1, 2, 3$, respectively [126]. The expression for TM WGM spectrum looks similar

$$k_{\nu,q} \simeq \frac{1}{a\sqrt{\epsilon_0}}\left[\nu + \alpha_q\left(\frac{\nu}{2}\right)^{1/3} - \frac{1}{\sqrt{\epsilon_0(\epsilon_0 - 1)}} + \frac{3\alpha_q^2}{20}\left(\frac{2}{\nu}\right)^{1/3} + O(\nu^{-2/3})\right]. \quad (8)$$

The third terms in the right hand side of (7) and (8) represent the fact that the dielectric WGRs are open resonators. The optical field tunnels outside the resonator's surface at the characteristic length $\sim 1/(k_{\nu,q}\sqrt{\epsilon_0 - 1})$. The larger the susceptibility ϵ_0, the smaller is this length and the closer are the solutions to the solution for a closed resonator.

The first order approximation for the mode eigenfunctions and eigenvalues may be found from the solution of an approximate equation

$$\frac{\partial^2 \Psi}{\partial r'^2} + \left(k^2 \epsilon_0 - \frac{\nu(\nu+1)}{a^2} - r'\frac{2\nu(\nu+1)}{a^3}\right)\Psi = 0 \quad (9)$$

where we assume that $\nu \gg 1, r' = a - r, \Psi(0) = \Psi(a) = 0$, and a is the radius of the sphere or cylinder. Comparison of the numerical solution of the exact (6) and of the approximate (9) shows that the solution of (9)

$$\Psi_q(r') = \Psi_{q,0} Ai\left[\left(\frac{2\nu(\nu+1)}{a^3}\right)^{1/3}\frac{r'}{\alpha_q} - \alpha_q\right] \quad (10)$$

where $\Psi_{q,0}$ is the field amplitude and k_q is the root of the equation

$$k_{\nu,q}^2 \epsilon_0 - \frac{\nu(\nu+1)}{a^2} = \alpha_q\left(\frac{2\nu(\nu+1)}{a^3}\right)^{2/3} \quad (11)$$

gives satisfactory results for the eigenvalues as well as eigenfunctions of the exact problem. For instance, it is easy to see that (11) gives a close approximation of the first two terms of the decomposition (7).

There are several experimental studies of WGM spectra in spherical WGRs. Spectra of liquid droplets was studied in [26], [27], [130]–[133]. High-resolution spectroscopy of WGMs in large fused silica spheres (3.8 cm diameter) was reported in [56]. Visualization of WGMs in WGRs was also realized [134]–[141]. The results of study of the microwave modes of cylindrical as well as spherical dielectric resonators was reported in [142], [143].

B. Spectral Properties of WGMs in a Spheroid

Calculations for spheroid WGRs are more complicated. Deviation of the resonator shape from ideal sphere results in removal of degeneracy by m and other effects. For a spheroid (ellipsoid) characterized with small deviation from an ideal sphere, the frequency shift of each WGM is given by [144]

$$\frac{\Delta k_{\nu,m,q}}{k_{\nu,q}(a)} = \frac{\zeta}{6}\left[\frac{3m^2}{\nu(\nu+1)} - 1\right] \tag{12}$$

where $\zeta = (r_{\rm pol} - r_{\rm eq})/a, r_{\rm eq}$, and $r_{\rm pol}$ are the equatorial and polar radii, respectively. Hence, $\zeta > 0$ corresponds to prolate, and $\zeta < 0$ corresponds to oblate ellipsoids, respectively. Equation (12) does not depend on number q and average radius $a \approx (r_{\rm pol} + 2r_{\rm eq})/3$ (we derive it from the condition of volume conservation if the sphere is transformed into a spheroid $r_{\rm eq}^2 r_{\rm pol} = a^3$).

Approximations for oblate and prolate spheroids with large eccentricity were reported in [117] and [145]–[147]. To estimate the eigenvalues of the WGMs in an oblate spheroid of large equatorial radius $r_{\rm eq}$, small polar radius $r_{\rm pol}, r_{\rm eq} \gg r_{\rm pol}$, and eccentricity $\tilde\zeta = \sqrt{1 - r_{\rm pol}^2/r_{\rm eq}^2}$, we recall that eigenfrequencies of high-order WGMs $(\nu \gg 1, m \simeq \nu)$ (in an ideal sphere as well as in the spheroid) can be approximated via solutions of the scalar wave equation with zero boundary conditions. For the spheroid expression, similar to (7), we have

$$\sqrt{\epsilon_0}\tilde k_{m,q} r_{eq} = T_{m,q} - \sqrt{\frac{\epsilon_0}{\epsilon_0 - 1}} \tag{13}$$

where $\tilde k_{m,q} = \sqrt{k_{\nu,m,q}^2 - k_\perp^2}, k_\perp$ is the wave number for the angular spheroidal function, and $T_{m,q}$ is the q-th zero for cylindrical Bessel function $J_m(T_{m,q}) = 0$. Since whispering gallery modes are confined in the cavity "equatorial" region we use cylindrical, not spherical, functions in our calculations.

For our purposes a rough approximation of $k_\perp$ is enough

$$k_\perp^2 \approx \frac{2(\nu - m) + 1}{r_{\rm eq}^2\sqrt{1 - \tilde\zeta^2}} m. \tag{14}$$

Because $k_{\nu,m,q} \approx \nu\epsilon_0^{-1/2} r_{\rm eq}^{-1}$ and $k_\perp \approx (1 - \tilde\zeta^2)^{-1/4}\nu^{1/2} r_{\rm eq}^{-1}$, $k_{\nu,m,q} \gg k_\perp$, we write

$$k_{\nu,m,q} \simeq \tilde k_{m,q} + \frac{k_\perp^2}{2\tilde k_{m,q}}. \tag{15}$$

Substituting (13) and (14) into (15), and taking into account that $T_{m,q} - [\epsilon_0/(\epsilon_0 - 1)]^{1/2} \approx k_{\nu,q}/(r_{eq}\epsilon_0^{1/2}) - (\nu - m + 1/2)$ $(k_{\nu,q}(r_{eq})$ is given by (7), we finally derive

$$k_{\nu,m,q} - k_{\nu,q}(r_{eq}) = \frac{2(\nu - m) + 1}{2r_{eq}\sqrt{\epsilon_0}}\left(\frac{1 - \sqrt{1 - \tilde\zeta^2}}{\sqrt{1 - \tilde\zeta^2}}\right). \tag{16}$$

To compare this expression with (12) we note that both (12) and (16) give the same the frequency splitting between two successive modes in the case of small eccentricity

$$\begin{aligned} k_{\nu+1,m,q} - k_{\nu,m,q} &\simeq k_{\nu,m,q} - k_{nu-1,m,q} \\ &\approx \frac{1}{\epsilon_0^{1/2} a}(1 + 0.62\nu^{-2/3} + O(\nu^{-5/3})) \end{aligned} \tag{17}$$

$$\begin{aligned} k_{\nu,m+1,q} - k_{\nu,m,q} &\simeq k_{\nu,m,q} - k_{\nu,m-1,q} \\ &= -\frac{1}{\epsilon_0^{1/2} r_{eq}}\left(1 - \sqrt{\frac{1 - \tilde\zeta^2}{\sqrt{1 - \tilde\zeta^2}}}\right). \end{aligned} \tag{18}$$

Spectra of highly oblate spheroid WGRs was studied experimentally in [146], [148].

C. "Engineering" of WGMs Spectra

Applications of WGMs call for efficient methods of engineering WGM spectra. Originally proposed spherical WGRs are overmoded [149] with complex quasi-periodic spectra and unequal mode spacings resulting from both material and cavity dispersion. There are several recipes for control of WGM spectra. Manipulations by the WGRs shape result in controllable rarefication of the resonators spectra [105]. As is shown above, significant reduction in the mode spectral density is achieved in highly oblate spheroidal WGRs [146], [148]. Microring resonators, as quasi-single-dimension objects, naturally have very clean spectra [100], [150], [151] consisting of quasi-equidistant fundamental modes corresponding to successive numbers of wavelengths packed along the perimeter of WGR.

It is not always practically useful or convenient to tune the spectrum of a WGR by changing its shape. This is especially related to large enough WGRs, having tens of gigahertz free spectrum range. The dense optical spectrum associated with a big WGR represents a limitation for its application in systems that require large sidemode rejection. The spectral density of the majority of geometries for WGRs could be significantly decreased with application of specially designed mode dampers [152]. A prism, or other polished piece of a material, with index of refraction higher than the index of refraction of the resonator material, can be used to decrease Q-factors of the majority of the unwanted modes of the resonator. Ideally, only the modes of the main sequence survive. Q-factor of those modes slightly changes in the mode rarefication process, but many applications tolerate that change. The selective suppression of the modes Q's is possible because various WGMs are localized in various geometrical places. To avoid deterioration of the Q-factors of the main mode sequence the damper should not be in the same plane as the coupling prism, i.e., angle a should not be equal to zero.

Another problem of management of WGM spectra is related to fabrication of a WGR with an equidistant spectrum. Performance and range of applications based on WGRs will be significantly expanded if a method is found to make resonator modes equally spaced with precision corresponding to a fraction of the resonance bandwidth in the frequency band corresponding to the high quality factors of the resonator modes. Such a dielectric resonator with equidistant mode spectrum is similar to the FP resonator with vacuum spacing between the mirrors. A WGR with equidistant spectrum may be used, e.g., in frequency comb generators, optical pulse generators, broadband energy storage circuits of electrooptical devices, and in other applications where conventional optical cavities are utilized.

Within current technology based on uniform resonator material, the smaller is the resonator size, the more the resonator geometrical dispersion is manifested in unequal spectral separation between adjacent modes. The problem is rooted in the fact that the radial distribution of WGMs is frequency dependent. Higher frequency modes propagate along paths that are slightly closer to the surface than those of lower frequency modes. Thus, higher frequency modes travel in trajectories of slightly larger radius and slightly longer optical path lengths.

The modal dispersion, for practical applications purposes, can be qualified by the ratio of modal bandwidth and the difference between the two frequency intervals separating a mode from its left and right neighbor, correspondingly. This number can be interpreted as a maximum number of exactly equidistant frequencies that can be fitted under the slowly expanding grid of WGM modes with finite bandwidth. This number, in the case of large ν, can be estimated as

$$N = \max_q \left| \left(\frac{\partial^2 k_{\nu,q}}{\partial \nu^2} \right)^{-1} \frac{k_{\nu,q}}{2Q} \right|. \quad (19)$$

From (7) we derive

$$N \simeq 1.2 \, \frac{\nu^{8/3}}{Q}. \quad (20)$$

If the Q is realistically high, modal dispersion can prevent coherent optical comb generation. For example, for $\nu = 10^3$ resonator modes can already be treated as un-equidistant for $Q \geq 10^8 (N \leq 1)$. Keeping in mind that maximum achieved quality factor for a WGM in fused silica resonators is 9×10^9 [45], one can see that the geometrical dispersion problem is really important in small WGRs. On the other hand, it is the material, not geometrical, dispersion that is the most significant in relatively large resonators having a few millimeters in diameter [153].

Optical path length is a function of both the physical distance and the index of refraction. One way of creating a WGR with an equidistant spectrum is by mutual cancellation of the material and geometrical dispersion [154] if an optimum size of the resonator is chosen. Another method is based on the fabrication of a resonator out of a cylindrically symmetric material whose index decreases in the radial direction [155]. With the proper choice of a gradient of the refractive index circular trajectories corresponding to WGM at different frequencies will have identical optical path lengths. This results in equidistant mode spectrum of the resonator [156].

D. Mode Volume

Basic properties of a WGR include its geometrical characteristics of the field localization (mode volume). The volume of WGMs is especially important for nonlinear applications of the resonators [97]. WGRs can have mode volumes orders of magnitude less than in Gaussian-mode resonators. The mode volume for a spherical WGR can be estimated as [97]

$$V = 3.4\pi^{3/2} \left(\frac{\lambda}{2\pi n} \right)^3 \nu^{11/6} \sqrt{\nu - m + 1} \quad (21)$$

where λ is the wavelength of the pumping light.

Numerical calculations of the mode volume of an arbitrary dielectric spheroid were presented in [157]. The additional confinement provided by the toroidal geometry results in the mode volume decrease compared with spherical geometry. In the case of oblate spheroid (toroid) when WGMs are localized near the largest spheroid circumference having radius a, the reduction of modal volume scales as $\sim (d/2a)^{1/4}$ with respect to that of a spherical cavity, where d is the minor (transverse curvature) diameter of the toroid. This is true if d exceeds the wavelength of the WGMs. For smaller diameters d, the spatial confinement becomes strong enough that the WGMs are additionally compressed in the radial direction. This results in a faster reduction of modal volume. The WGM's transverse structure becomes identical to that of a bare single mode fiber closed into a loop.

The comparative characteristics of volumes of WGMs in various kinds of WGRs can be found in [8], [157], [158]. WGMs with the smallest mode volumes, close to a cubic wavelength, was realized in photonic crystal resonators [159], [70], disordered media resonators [160], microdisks [161], [162], and microrings [163]. However, those tiny resonators have much smaller Q-factors than their "mesoscale" WGR counterparts (few hundred micron in diameter and larger).

E. Q-Factor

Amongst many parameters that characterize the resonator the Q-factor is a basic one. The Q-factor is related to the lifetime of light energy in the resonator mode (τ) as $Q = \omega\tau$, where ω is the frequency of the mode. The ring down time corresponding to a mode with $Q = 10^{10}$ and wavelength $\lambda = 1.3$ μm is 7μs, thus making ultrahigh Q resonators potentially attractive as light storage devices.

Q-factor is fundamentally restricted by the radiative emission, which is unavoidable in open dielectric resonators. This emission generally is negligible. As was pointed out in [15], the radiative decay results in $Q = 2.7 \times 10^7$ for $\lambda/a = 0.571$, $\epsilon_0 = 5$, and $\nu = 20$. The value of the quality factor determined by the radiative decay becomes enormous in larger resonators. For example, it is $Q = 10^{73}$ at $\lambda = 0.6$ μm for a water drop ($\epsilon_0 = 1.77$) having radius $a = 50$ μm [6].

The highest Q-factor up to the date, $Q = 2 \times 10^{10}$, was achieved in crystalline WGRs [104]. The oblate spheroidal resonators with diameter 0.5–12 mm and thickness 0.03–1 mm were fabricated out of single-crystal blocks by standard diamond cutting and lapping, and optical polishing techniques. It is believed that the current Q-factor values are limited by extrinsic losses in particular specimens of material (originating from uncontrolled residual doping nonstoichiometry). As already mentioned above, values of Q-factor can reach 10^{13} at 1.55 μm in fluorite resonators.

The highest measured Q-factor in amorphous WGM resonators is $Q = 8 \times 10^9$ at 633 nm [45]. Approximately the same WGM Q-factors were observed in near infrared [46] (1.55 μm) and at 780 nm [164]. The near-spherical WGR radius varied from 60 μm to 200 μm in [164], and from 600 to 800 μm in [45], [46]. The measured Q-factors were close to the maximum achievable value for the fused silica at 630 nm [47].

Q-factors measured in liquid WGRs (e.g., free-flying or trapped droplets of liquid aerosols) are less than 10^5. The problem is in difficulty of excitation and detection of WGMs with larger Q using free beam technique [165]. Theoretical implications from the experimental data for the Q factors in liquid WGRs are more optimistic: $Q \geq 10^6$ [41], [166]–[169] for $\sim$20 μm droplets. On the other hand, it was shown that a pendant 400 μm liquid-hydrogen droplet can achieve high-Q values that exceed 10^9 for WGMs in the ultraviolet [170].

Quality factor $Q \approx 2 \times 10^8$ at $\lambda = 2.014$ μm ($\alpha \leq 5 \times 10^{-4}$ cm^{-1}) was reported earlier for a multiple total-internal-reflection resonator, analogous to a WGR, used in optical parametric oscillators pumped at 1064 nm [118]. The same order of Q was reported for fused silica resonators of the same shape at 1064 nm [171].

Quality factors of microring and microdisk WGRs typically do not exceed 10^5. For instance, all epitaxial semiconductor 10 μm diameter microring resonators vertically coupled to buried heterostructure bus waveguides have $Q = 2.5 \times 10^3$ [172]. Unloaded Q-factor of the order of 10^5 was demonstrated in 80 μm microdiscks [33]. Micron-size microdisk semiconductor resonators have Qs of order of 10^4 [161], [162].

VI. Nonlinear Properties of WGMs

Whispering gallery modes play a significant role in modern nonlinear optics. High quality factors and large field densities, associated with WGMs in dielectric resonators, result in resonant enhancement of nonlinear interactions of various kinds [45], [46], [97], [164], [173], [174]. These modes provide the opportunity to achieve a high nonlinear response with relatively weak electromagnetic fields, even if the cavity is fabricated from a material with low nonlinearity, as is usually the case for optically transparent materials.

Crystalline WGRs with kiloHertz-range resonance bandwidths at room temperature and high resonance contrast (50% and more) are extremely promising for integration into high performance optical networks. Because of small modal volumes and extremely narrow single-photon resonances, a variety of low-threshold nonlinear effects can be observed in WGRs based on small broadband nonlinear susceptibilities. As an example, let us consider the thermo-optical instability in WGRs.

Thermal nonlinearity is important in high-Q WGRs [175]–[178]. Thermorefractive coefficient for calcium fluoride, for instance, is $\beta = n_0^{-1}\partial n/\partial T \simeq -1 \times 10^{-5}/K$. It means that the frequency ω of a WGM increases by $10^{-5}\omega$ if the temperature T increases by one degree Kelvin (follows from $\omega \approx c\nu/an_0(1+\beta)$, where c is the speed of light in the vacuum, $\nu \gg 1$ is the mode number, a is the radius of the resonator, and n_0 is the index of refraction). This shift is five orders of magnitude larger than the width of the resonance if $Q = 10^{10}$. Given extremely small modal volume, even if a small fraction of optical power is absorbed, it may be enough to "heat the WGR out of resonance."

The immediate manifestation of this effect is observation of non-Lorentzian and hysteretic resonances during tunable laser spectroscopy of WGMs, with lineshapes depending on the input optical power, and the speed and direction of the laser frequency sweeping [176]. If WGMs are probed by frequency-swept laser light for measurement of the Q-factor from the observed resonance curves, it is important to choose input power small enough and scan fast enough to minimize the pulling effect of thermal nonlinearity and ensure Lorentzian lineshapes.

VII. Conclusion

In this paper we review basic properties of WGRs with emphasis on their applications. The basic distinctive property of the resonators is that they do not need mirrors. The light is confined in the resonators due to the total internal reflection. The modes of the resonators could have extremely high quality factors and small volumes. The spectra of the resonators could be engineered by changing shape of their surface. The resonators could operate in a wide optical band determined solely by the material the resonators are made of. The resonator properties are robust with respect to the environment. We show that existing technology of the fabrication of and manipulations with the resonators is developed well enough to expect steady growth of the applications of the resonators not only in selected scientific fields, but also in everyday life.

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