Parametric oscillations in a whispering gallery resonator

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Received July 28, 2006; revised October 13, 2006; accepted October 13, 2006; posted October 17, 2006 (Doc. ID 73566); published December 23, 2006

We demonstrate strongly nondegenerate optical continuous-wave parametric oscillations in crystalline whispering gallery mode resonators fabricated from $LiNbO_3$. The required phase matching is achieved by geometrical confinement of the modes in the resonator. © 2006 Optical Society of America

OCIS codes: 190.4410, 190.4360, 230.5750.

Whispering gallery mode (WGM) resonators are used for the study of various nonlinear optical phenomena at low optical intensities as well as for the fabrication of novel optical devices. Low-threshold Raman lasing,[1] optomechanical oscillations,[2] and frequency doubling,[3] as well as hyperparametric oscillations[4,5] based on these resonators have been recently demonstrated. In this Letter we report the observation of low-threshold, strongly nondegenerate parametric oscillations in lithium niobate WGM resonators.

The principle of operation of our oscillator is rooted in two phenomena observed earlier: (i) stimulated Raman scattering by polaritons in lithium niobate[6] and (ii) phase matching of nonlinear processes via geometrical confinement of the light.[7] Our oscillator has certain similarities with terahertz-wave oscillators based on $LiNbO_3$ crystals.[8] The key difference is that the novel geometrical configuration of our system allows oscillation in the continuous-wave regime. We show that the geometry of a WGM resonator results in the modification of the oscillation frequency. High quality factors of optical WGMs lead to a much lower oscillation threshold compared with the threshold reported in Ref. 8.

In what follows we show theoretically that the proper morphology of a WGM resonator made of lithium niobate results in triply resonant phase-matched parametric process. We predict the threshold and frequency of the parametric oscillation. To confirm these theoretical predictions, we fabricated a set of disk-shaped crystalline WGM resonators and demonstrated the parametric oscillation experimentally.

For the sake of simplicity we assume that each field involved in the parametric interaction (signal, idler, and pump) is resonant with only one WGM. The electric fields of the pump, signal, and idler WGMs can be represented as $E_{(l)} = \Psi_l [2\pi\hbar\omega_{(l)}/\epsilon_{(l)} V_{(l)}]^{1/2} a_{(l)} \exp[-i\omega_{(l)} t] + \text{c.c.}$, where $\Psi_l = \bar{\Psi}_l \sin(k_{m(l)} z) J_{\nu(l)}(k_{\nu(l),q(l)} r) \exp(i\nu_l \phi)$ with $\bar{\Psi}_l$ a scaling factor; $a_{(l)}$ and $a^\dagger_{(l)}$ are annihilation and creation operators for the mode; $l=\{p,s,i\}$ is the index that stands for the pump, signal, and idler, respectively; ϕ, r, and z are the coordinates; $\nu_{(l)}=0,1,2,\ldots$, $m_{(l)}=1,2,\ldots$, and $q_{(l)}=1,2,\ldots$ are the angular, longitudinal, and radial quantization numbers; $V_{(l)} = \int_V |\Psi_{(l)}(r)|^2 dV$ is the mode volume; $k_{m(p,s)} = \pi m_{(l)}/L$ and $k_{\nu(l),q(l)} a \simeq \nu(l) + \alpha_{q(l)} [\nu(l)/2]^{1/3}$ are the longitudinal and transverse wave numbers for which the characteristic equation $k^2_{m(l)} + k^2_{\nu(l),q(l)} = k^2_{(l)} \epsilon_{(l)}$ holds; L is the height of the resonator; $\epsilon_{(l)}$ is the susceptibility of the resonator material at the pump–signal–idler frequencies; a is the resonator radius; and α_q is the qth root of the Airy function, $\mathrm{Ai}(-z)$.

The parametric interaction takes place if the energy conservation law and phase-matching condition are fulfilled: $\omega_{(p)} = \omega_{(s)} + \omega_{(i)}$ and $V_o = \int_V \Psi_p \Psi_s \Psi_i dV \neq 0$. Let us show that there are signal and idler frequencies such that the phase-matching conditions are satisfied and estimate the oscillation frequencies for an oscillator based on a $LiNbO_3$ WGM resonator that is 0.087 cm in diameter and $L=50$ μm in thickness. We consider, for the sake of simplicity, a nondegenerate parametric interaction of WGMs such that the frequencies of the ordinarily polarized pump $[\omega_{(p)}]$ and signal $[\omega_{(s)}]$ modes exceed $2\pi \times 100$ THz, while the frequency of the extraordinarily polarized idler field $[\omega_{(i)}]$ is less than the frequency of the lowest transverse TE phonon branch ($2\pi \times 7.44$ THz) of $LiNbO_3$.

The resonator is pumped at a WGM belonging to the basic TM mode sequence, i.e. $m_{(p)}=1$ and $q_{(p)}=1$. Let us assume that a signal TM mode with $m_{(s)}=1$ and $q_{(s)}=1$ oscillates and find the parameters of the TE idler mode. The phase-matching conditions are fulfilled if $m_{(i)}=1$, $\nu_{(i)} = \pi a \epsilon^{1/2}_{(p)} / [L(\epsilon_{(i)} - \epsilon_{(p)})^{1/2}]$, and $\omega_{(i)} = \pi c / [L(\epsilon_{(i)} - \epsilon_{(p)})^{1/2}]$. This is the lowest-frequency idler mode that sustains phase-matching conditions. Its frequency is equal to $\omega_{(i)} = 2\pi \times 652$ GHz for the given resonator shape. Let us stress here that $\nu_{(i)}$ is an integer. Hence the radius of the resonator should be properly adjusted to meet the phase-matching condition.

We find the threshold of the oscillations starting with the interaction Hamiltonian in the slowly varying amplitude and phase approximation, which is given by

$$H = \hbar g (a^\dagger_s a^\dagger_i a_p + a^\dagger_p a_s a_i), \tag{1}$$

where the coupling constant is $g = [\chi^{(2)} V_o/\hbar] \Pi_l [2\pi\hbar\omega_{(l)}/\epsilon_{(l)} V_{(l)}]^{1/2}$.

Using this Hamiltonian, we derive equations of motion

$$\dot{a}_{(p)} = -\gamma_{(p)} a_{(p)} - i g a_{(s)} a_{(i)} + F, \tag{2}$$

$$\dot{a}_{(s)} = -\gamma_{(s)} a_{(s)} - i g a_{(i)}^{\dagger} a_{(p)}, \tag{3}$$

$$\dot{a}_{(i)} = -\gamma_{(i)} a_{(i)} - i g a_{(i)}^{\dagger} a_{(p)}, \tag{4}$$

where $\gamma_{(l)}$ is the WGM decay rate. Parameter F describes pumping from outside of the system. The external pumping is described by the expression $|F|^2/\gamma_{(p)}^2 = P_{(p)} Q_{(p)}/(\hbar\omega_{(p)}^2)$, where $Q_{(p)} = \omega_{(p)}/(2\gamma_{(p)})$ is the mode quality factor and $P_{(p)}$ is the pump power.

Solving Eqs. (2)–(4) in steady state, we obtain

$$|a_s|^2 = \frac{\gamma_{(i)}\gamma_{(p)}}{g^2}\left(\frac{g}{\sqrt{\gamma_{(i)}\gamma_{(s)}}}\frac{|F|}{\gamma_{(p)}} - 1\right), \tag{5}$$

$$\gamma_{(s)}|a_{(s)}|^2 = \gamma_{(i)}|a_{(i)}|^2, \tag{6}$$

which results in the threshold condition for the parametric oscillation,

$$P_{(p)} \geqslant \frac{2\omega_{(p)}}{[\chi^{(2)}]^2 V_o^2}\prod_l \frac{\epsilon_{(l)} V_{(l)}}{4\pi Q_{(l)}}. \tag{7}$$

To estimate the oscillation threshold we assume that $\lambda_{(p)} = 1.32\ \mu\text{m}$ $[\omega_{(p)} = 1.43\times 10^{15}\ \text{s}^{-1}]$, $V_o \simeq V_{(s)} \simeq V_{(p)} \simeq 2\pi a^2 L \nu_{(p)}^{-2/3} = 3\times 10^{-7}\ \text{cm}^3$, $\epsilon_{(p)} = \epsilon_{(s)} = 4.8$ (TM modes), $\epsilon_{(i)} = 26$ (TE mode), $\chi^{(2)} = 10^{-7}$ cgs, $Q_{(s)} \simeq Q_{(p)} \simeq 10^8$. We also should find the quality factor as well as the volume of the idler mode. The quality factor can be estimated from $Q_{(i)} = 2\pi\epsilon_{(i)}^{1/2}/[\lambda_{(i)}\alpha_{(i)}]$, where the inverse absorption length is $\alpha_{(i)} \approx \epsilon_{(i)}^{1/2}\omega_{(i)}^2/(\Gamma c)$. Parameter Γ is determined by the parameters of the low-frequency polaritonic branch of $LiNbO_3$ and can be approximated by $\Gamma \approx 9\times 10^{14}\ \text{s}^{-1}$, using measurement results reported in Ref. 8. The quality factor for the idler field with frequency $\omega_{(i)} = 2\pi\times 652$ GHz is then $Q_{(i)} = \Gamma/\omega_{(i)} \simeq 200$. The mode volume for the idler field is $V_{(i)} \simeq 10^{-5}\ \text{cm}^3$. Combining all the parameters, we infer $P_p \simeq 30$ mW. This value of the threshold power can be observed experimentally.

The lowest threshold predicted for nondegenerate parametric oscillation involves the lowest-frequency idler mode. Optical parameters of the system are minimally dependent on idler frequency, while quality factor and volume of the idler mode significantly depend on its frequency. As a result the threshold power is proportional to $\omega_{(i)}^{1/3}$ for the case of small $\omega_{(i)}$. The dependence is much faster when $\omega_{(i)}$ approaches $2\pi\times 7.44$ THz.

We have considered the case in which the thickness of the WGM resonator L exceeds the idler wavelength, so all the fields involved in the oscillation are resonant with the WGM modes. The oscillation is also possible if the wavelength of the idler field in the material is larger than the thickness of the resonator, so the field cannot be confined or quantized in the z direction. In this case the equivalent quality factor $\tilde{Q}_{(i)}$ of the idler is determined by the interaction time $\tau_{(i)}$ of the idler photon with the probe and signal photons confined in the resonator, such that $\tilde{Q}_{(i)} \sim \tau_{(i)}\omega_{(i)}$. If the idler is a running wave, we find $\tau \sim \epsilon_{(i)}^{1/2} L/c$ and, hence, $\tilde{Q}_{(i)}$ increases with the frequency increase. The equivalent quality factor of the unconfined filed is less than the quality factor of confined modes for the above specified numerical parameters, so the oscillation with the idler corresponding to the lowest-order phase-matched WGM has the lowest threshold.

We have shown theoretically the possibility of nondegenerate oscillation in a crystalline WGM resonator. In what follows we describe the experiment that confirms this theoretical prediction. Our experimental setup is shown in Fig. 1. We repeated the experiment with two different pump lasers: Nd:YAG (Lightwave Electronics), centered at 1319 nm, maximum 100 mW output at the output of the polarization-maintaining fiber, and 4 kHz linewidth; as well as fiber laser (Koheras) centered at 1559 nm, maximum 150 mW power output at the output of the fiber, and 2 kHz linewidth. The laser light was sent through a polarization controller, the collimator (coated grin lens collimators, 0.25 pitch, 1.8 mm in diameter), and a diamond prism coupler ($1.5\ \text{mm}\times 1.5\ \text{mm}\times 1.5\ \text{mm}$, type IIa diamond), to a WGM resonator made from lithium niobate. The pump polarization is selected such that the TM mode family is excited. The coupling efficiency was approximately 60% at both wavelengths.

The resonator is a disk with radius $a = 0.5$ mm and thickness 50 μm fabricated from a commercial grade flat Z-cut stoichiometric $LiNbO_3$ wafer (Oxide, Inc). The side wall of the disk is polished (see Ref. 9 for the fabrication details). We used three resonator preforms characterized with different rim thicknesses. Resonator I has a 10 μm wide rim with $\beta = 45°$ top and bottom degrees edges, resonator II has a 50 μm wide rim with $\beta = 12°$, and resonator III has an approximately 3 μm wide rim and $\beta = 45°$. Such shaping of the preforms changes the effective thickness of the resonators, allowing phase-matching conditions for signal and idler waves with different frequencies to be fulfilled. The maximum intrinsic quality factors of the TM modes of the resonators are $Q = 3.4\times 10^8$

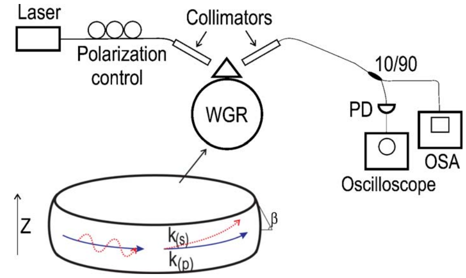

Fig. 1. (Color online) Experimental setup. The pump photon with wave vector $k_{(p)}$ scatters to signal photon $k_{(s)}$ and idler photon $k_{(i)}$. The idler photon is not necessarily localized in a WGM. The pump photon has TM polarization; i.e., its electric field is orthogonal to the axis of the resonator.

(1319 nm) and $Q = 4.2 \times 10^8$ (1559 nm). The loaded quality factor was approximately $Q = 7 \times 10^7$.

The light from the resonator was collected in a single-mode fiber and sent to a slow photodiode and oscilloscope to observe the WGM spectrum, as well as to an optical spectrum analyzer. The optical spectrum analyzer (Yokogawa AQ6319) has a resolution of 1.2 GHz at 1559 nm and 8.7 GHz at 1319 nm, high enough to observe the optical signal generated in the parametric process.

Tuning the power of the pump light, we detected several optical sidebands generated in the resonators [Figs. (2) and (3)]. The frequency of the low-frequency sidebands [$\omega_{(i)} < 7.44$ THz] depends on the shape of the resonator through the phase-matching conditions. The light generated in the thicker resonator (I) has lower frequency. The threshold power for the sideband observation was approximately 20 mW for a 0.76 THz idler [see Fig. 3(a)]. The Raman line at 7.44 THz was not observed for the given pump power. This confirms that we have observed the strongly nondegenerate parametric oscillation predicted theoretically.

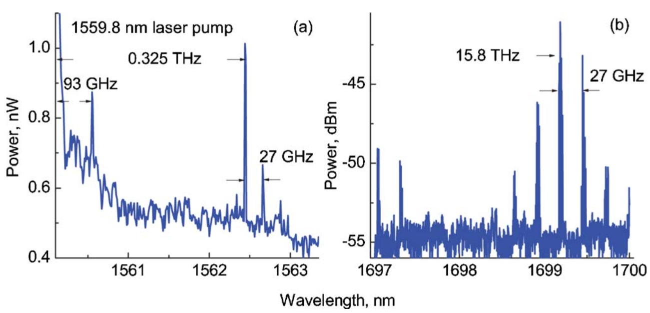

Fig. 2. (Color online) Stokes sidebands generated in resonators I and resonator III illuminated with 1559 nm light.

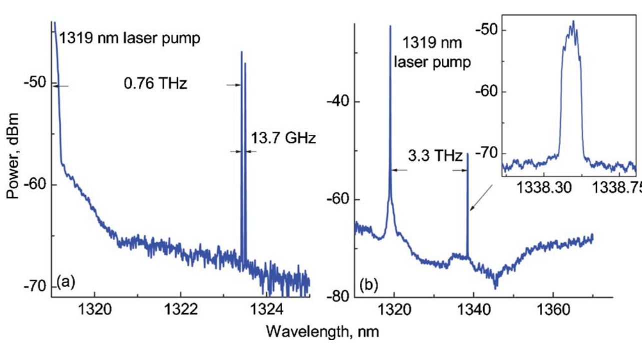

Fig. 3. (Color online) Stokes sidebands generated in resonator (a) I and (b) II, illuminated with 1319 nm light. The frequency shift of the signal wave is bigger in the thinner resonator. The 13.7 GHz splitting between the signal modes is less than the free spectral range of the resonator.

We also observed oscillation with confined and unconfined idlers. For instance, Fig. 2(a) illustrates the case of the unconfined idler. The wavelength of the idler light, inferred from the frequency difference of the observed signal and pump light, apparently exceeds the thickness of the resonator.

We observed forward scattering on the 628 cm^{-1} (18.8 THz) polariton branch predicted theoretically in Ref. 10 [see Fig. 2(b)]. The frequency shift of the signal (15.8 THz) approximately corresponds to the phase-matching conditions for the polariton in our geometry. Realizing a proper coupling scheme for the terahertz light would allow its direct observation, similar to the observation in.[8] Hence, this oscillator is promising as a source of continuous wave terahertz radiation.

In conclusion, we have theoretically predicted, and experimentally demonstrated, strongly nondegenerate parametric oscillation in whispering gallery mode resonators made from lithium niobate. The phase-matching condition for the oscillation is satisfied by the morphology of the system.

The research described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration and partial sponsorship from Defense Advanced Research Projects Agency Analog Optical Signal Processing program. A. Matsko's e-mail address is Andrey.Matsko@jpl.nasa.gov.

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